A Nonlinear Galerkin Scheme Involving Vector and Tensor Spherical Harmonics for Solving the Incompressible Navier-Stokes Equation on the Sphere

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Abstract:

The first part of this talk is dedicated to the introduction of the nonlinear Galerkin method applied to the Navier-Stokes equation on the rotating sphere. The Navier-Stokes equation plays a fundamental role in meteorology by modelling meso-scale (stratified) atmospherical flows. The nonlinear Galerkin method is implemented by using type three vector spherical harmonics, and convergence is sketched. Further, it is shown that the occurring coupling terms involving up to three vector spherical harmonics in the nonlinear advection term can be expressed algebraically in terms of Wigner-3j coefficients. To improve numerical efficiency we introduce an exact FFT-based pseudo spectral algorithm for computing the Fourier coefficients of the nonlinear advection term. The resulting method scales with O(N^3), if N denotes the maximal spherical harmonic degree. Finally, an extensive numerical simulation for moderate Reynold's numbers is presented