ARMA: AutoRegressive Moving Average

Model Identification Criteria

- Objective methods (specified decision criteria)
- Subjective methods (statistical tests, graphics, etc.)

Modeling temporal correlations of GNSS observations (motivation)

- Analyzing physically correlated processes (e.g., in geosciences)
- Generating prognostic models (e.g., in economic sciences)

ARMA Applications

- Generating prognostic models (e.g., in economic sciences)

ARMA(p,q) model

\[ \phi(B) \theta(B^{-1}) Z_t = c + \epsilon_t \]

ARIMA(p,d,q) model

\[ (1-B)^d Z_t = \phi(B) \theta(B^{-1}) \epsilon_t \]

Introduction

Different model identification criteria

ARMA modeling of GNSS residual using different model identification criteria

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Criterion AICC (Akaike 1973)

\[
AICC = \frac{(b + 1 + d) \ln n + b + d}{n} + \frac{1}{n} \frac{b + d}{b + 1 + d} \sum_{j=1}^{n} (z_{j} - \hat{z}_{j})^2
\]

\[
\hat{z}_{j} = y_{j} - \hat{\phi} \sum_{k=1}^{p} \hat{y}_{j-k} - \hat{\theta} \sum_{k=1}^{q} \hat{\theta}_{j-k}
\]

Assumptions and Prior Conditions

1. Gaussian likelihood function of an ARMA(p,d,q) process
2. Kalman filter of Gaussian process, e.g.
3. Maximum likelihood estimator of \( \phi \) is an ARMA(p,d,q) process
4. One-step predictor for \( x_{j} \)
5. Rescaled mean squared errors of \( \hat{z}_{j} \)

Kullback-Leibler Index (KLI)

\[
\Delta(\hat{\phi}, \phi) = \ln \left( \frac{L(\hat{\phi}, \hat{\theta})}{L(\phi, \theta)} \right) = \sum_{j=1}^{n} (z_{j} - \hat{z}_{j})^2
\]

Using large-sample approximations:

\[
u = \frac{v}{\sum_{j=1}^{n} (z_{j} - \hat{z}_{j})^2}
\]

Estimation of KLI

\[
u = \frac{v}{\sum_{j=1}^{n} (z_{j} - \hat{z}_{j})^2}
\]

Criterion AIC (Akaike 1973)

\[
AIC = (b + d) \ln n + \frac{b + d}{n} \sum_{j=1}^{n} (z_{j} - \hat{z}_{j})^2
\]

Using large-sample approximations:

\[
u = \frac{v}{\sum_{j=1}^{n} (z_{j} - \hat{z}_{j})^2}
\]
Criteria CIC and GIC

CIC for AR(p) order selection (Klees and Broersen 2002)

\[
\prod \sum = pk \quad \text{CIC} \\
\text{with the residual (estimated WN) variance}
\]

\[
\hat{\sigma}^2 \quad \text{Yule-Walker and Burg estimates}
\]

CIC: Combined Information Criterion; GIC: Generalised Information Criterion

\[
\frac{b + d}{w} = \min \left( \text{ARMA}(p,d) \right)
\]

\[
\frac{b}{w} = \min \left( \text{ARMA}(p,d) \right)
\]

Model identification based on prediction error (PE)

\[
\frac{b}{w} = \min \left( \text{ARMA}(p,d) \right)
\]

Time series data

ARMA modeling using different identification criteria

Tab. 1: GPS processing strategies and data characteristics

Case study: data base

Fig. 1: SAPOS® network in the area of the state of Baden-Württemberg (Southwest Germany)

Fig. 2: Multipath (MP): Strong (HEDA); weak (other baselines)

Tab. 2: ARMA modeling using different identification criteria

SNR: Signal-to-Noise Ratio, MF: Mapping function
Fig. 2: Comparison of order selection using different model identification criteria

Fig. 3: Comparison of autocorrelation function (ACF) using different model identification criteria

Autocorrelation function (ACF)
Fig. 5: Comparison of ARMA simulation based on different model identification criteria.

- **Satellite pair:** PRN 0917
- **Site:** Tauberbischofsheim
- **ARMA simulation:**
  - **SDDR data:** TAAF1826168 (MP: weak)
    - **Simulation (AICC):** ARMA(3, 3)
    - **Simulation (CIC, GIC):** ARMA(2, 1)
  - **SDDR data:** HEDA1826168 (MP: strong)
    - **Simulation (AICC):** ARMA(5, 10)
    - **Simulation (CIC, GIC):** ARMA(3, 2)
Factors impacting the identification performance

- GNSS observational data (e.g., data quality, sample size)
- Applied algorithms for parameter estimation
- The highest candidate order for selection
- Higher selected orders with strong variability
- Lower selected orders with comparable modelling results
- Criterion AIC, CIC
- Criterion AICC
- Time-consuming computation
- Better performance in the case of low-quality data
- Similar within this case study
- Less computational cost
- Reasonable reduction of order search area

Comparison of the used identification criteria

The project "Improving the stochastic model of GPS observations by modelling physical correlations" (HE 1433/16-1/2) is supported by the Deutsche Forschungsgemeinschaft (DFG).

Questions & comments

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Thank you very much for your attention!