Multi-Resolution Ionospheric Attenuation for Single Frequency Receivers

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Mobile Multi-sensor Research Group
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Outline

- Motivation
- Wavelet Transform (WT)
- Error Behavior
- Wavelet De-Noising
- Wavelet 2 Steps De-Noising
- Data Collection and Analysis
- Conclusions
Motivation

GPS Errors
- High
- Medium
- Low

Frequency spectrum

Multipath
Cycle Slip
Noise
Iono/Tropo

Errors
- Multipath
- Cycle Slip
- Noise
- Iono/Tropo
Motivation

Evaluate the performance of Multi-Resolution Techniques in Ionosphere error reduction for single frequency user.

Double Difference Carrier Phase

\[
\Delta \nabla \Phi = \Delta \nabla \rho + \Delta \nabla d \rho + \lambda \Delta \nabla N - \Delta \nabla d_{\text{ion}} + \Delta \nabla d_{\text{trop}} + \Delta \nabla M_{\text{c}} + \varepsilon \Delta \nabla \Phi
\]

\[
\Delta \nabla P = \Delta \nabla \rho + \Delta \nabla d \rho + \Delta \nabla d_{\text{ion}} + \Delta \nabla d_{\text{trop}} + \Delta \nabla M_{\text{p}} + \varepsilon \Delta \nabla p
\]

Carrier phase measurement

Geometric range

Orbital error

Code multipath

Troposphere error

Carrier phase multipath

Code noise

Carrier phase noise

Double Difference

Integer Ambiguity

Ionosphere error

Very Hard to reach any reduction.

True geometry and low frequency Ionosphere interfere with each other. Hard to separate.
Wavelet Transform (WT) ...1/2

The wavelet transformation of a time-domain signal is defined in terms of the projections of this signal on to a family of functions that are all normalized dilations and translations of a wavelet function.

\[
CWT(m, n) = \int_{-\infty}^{\infty} X(t) \frac{1}{\sqrt{|m|}} \psi\left(\frac{t - n}{m}\right) dt
\]

- Wavelet coefficient
- Wavelet Function
- Scale
- Position or shift
Wavelet Transform (WT) ...

Multi-Resolution analysis
Pyramid Algorithm

\[ X(t) = \sum_{m \in \mathbb{Z}} a_n^{(m)} \Phi_{m,n}(t) + \sum_{m \in \mathbb{Z}} \sum_{n \in \mathbb{Z}} d_n^{(m)} \psi_{m,n}(t) \]

- **X(t)**: Signal
- **a**: Approximation
- **d**: Detail
- **m**: Scale
- **n**: Translation

Wavelet function

Scale

Mobile Multi-Sensor Systems Research Group
Methodology and Error Behavior

Original DD signal for satellite 14-31

Scalogram of DD signal

DD signal after removing polynomial of order 8

Scalogram of DD signal after polynomial removal

Remove by Polynomial fitting
Methodology and Error Behavior 2/3

DD Error for satellite 14-31

Scalogram of DD Error

DD signal after removing polynomial of order 4

Scalogram of DD signal after polynomial removal
Methodology and Error Behavior

**Short Baseline: Static (15 sec interval)**

- High Multipath and Noise 0.05 – 0.0015 Hz Scale 2–32
- Low Multipath 0.005 – 0.0015 Hz Scale 8–32
- Ionosphere canceled or Ignored

**Long Baseline: Static (15 sec interval)**

- High Multipath and Noise 0.05 – 0.0015 Hz Scale 2–32
- High Ionosphere 0.05 – 0.005 Hz Scale 2–8
- Low Multipath 0.002 – 0.001 Hz Scale 16–64
- Low Ionosphere $10^{-4} – 10^{-6}$ Hz Scale 64–512

Hard to separate from geometry
Wavelet De-Noising

Wavelet thresholding

Thresholding aims to reduce or remove the noise from a signal by setting some of the wavelet coefficients to zero.

Hard thresholding

\[
\hat{d}_n^m = \begin{cases} 
  d_n^m, & \text{if } |d_n^m| \geq \delta \\
  0, & \text{otherwise}
\end{cases}
\]

Median thresholding estimator

\[
\delta = \frac{\text{Median}\{|\langle x, \psi_{1,m} \rangle|\}_{m = 0,\ldots,N/2}}{0.6745}
\]

Where the values of \(\langle x, \psi_{1,m} \rangle\) is assumed to be normal distributed

Choosing the correct value of the threshold is a very important to avoid over smoothing or under smoothing.
Four stations at both University of Calgary and the Southern Alberta Network (SAN).

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Data Collection and Analysis ... 2/4

- L1 Solution
  - De-Noising
  - Wavelet L1 Float Solution Median
- L1 Solution
  - 2 Steps De-Noising
  - Wavelet L1 Float Solution Kill Details
- L1 Solution
  - 2 Steps De-Noising
  - Wavelet IF Float Solution Kill Details

Bias = L1 or L1/L2 solution – Reference (Bernese)
Percentage bias reduction = Bias - Bias_L1
## Data Collection and Analysis ... 3/4

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Percentage bias reduction when using wavelet techniques for GPS error mitigation
## Data Collection and Analysis ... 4/4

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mm

Standard deviation when using wavelet techniques for GPS error mitigation
Conclusions

- Low frequency Ionosphere is hard to separate from true geometry and looks like bias.
- De-noising using Median estimator is not efficient for Ionosphere error reduction it can only correct for tp10%.
- Two steps de-Noising can efficiently remove about 35% of the error in case of long baseline when using L1 frequency receiver and in case of Ionosphere free solution the reduction can reach (60-70%).
Acknowledgements

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