A correction model for zenith dry delay of GPS signals using regional meteorological sites

Xiaoguang Luo

Motivation

- **GPS-based determination of atmospheric water vapour**
  - Based on delay estimation of GPS signals
  - Zenith total delay (ZTD)
    - Estimated within GPS data processing
    - Site-specific troposphere parameters
    - Temporally variable (parameter time span, constraints)
  - Zenith dry delay (ZDD)
    - Calculated using meteorological (MET) data near GPS sites
    - Near-site MET data: observations, numerical weather model
    - Highly correlated with pressure of dry air
  - Zenith wet delay (ZWD): ZTD-ZDD → water vapour content

- **Problem: non-availability of near-site MET data**
Correcting a priori ZDD

\[ \text{ATM}_{\text{STD}}(T_0, p_0, \text{rh}_0) \]
\[ H_0 = 0 \text{ m} \]
\[ \text{Extrapolation} \]
\[ (T_S, p_S, \text{rh}_S) \]
\[ \text{A priori ZDD} (H_S) \]
\[ \text{invariable in time} \]
e.g., Berg (1948)
\[ \text{Correction value} \]
\[ \Delta ZDD_S = F(H_S) \]

Site altitude \((H_S)\) above MSL \((H_0)\)

Regional MET sites (e.g., DWD)
\[ \text{MET}_M(T_M, p_M, \text{rh}_M) \]
\[ \text{measurements} \]
\[ \text{ZDD} (\text{MET}_M) \]
using MET data
\[ \text{A priori ZDD} (H_M) \]
\[ \text{extrapolation} \]
\[ \Delta ZDD_M = \text{ZDD}(\text{MET}_M) - \text{ZDD}(H_M) = F(H_M) \]

Fig. 1: Schematic illustration of a correction model for a priori ZDD using regional meteorological sites

MSL: mean sea level; Index S/M: GPS/meteorological sites; DWD: Deutscher Wetterdienst

Data base

Fig. 2: Selected DWD meteorological stations and GPS sites with MET data in the area of southwest Germany
(digital elevation model: ETOP01, Amante and Eakins 2009)
A linear regression model

- Linear relationship between $H_M$ and $\Delta ZDD_M$

\[
\Delta ZDD_M = ZDD(MET_M) - ZDD(H_M)
\]

\[
\Delta ZDD_M = a \cdot H_M + b, \quad a, b: \text{regression coefficients}
\]

- Estimation of regression coefficients
  - Classical ordinary least-squares estimation (OLSE)
    - Minimising the squared sum of
      \[
      v : = (a \cdot H_M + b) - \Delta ZDD_M
      \]
    - Outlier detection based on studentised residuals $r_i(i)$
      \[
      r_i(i) = \frac{y_i}{\hat{\sigma}} = \frac{y_i}{\hat{\sigma}_i Q_{yy}(i,i)} \sim t_f
      \]
      \[
      T(i) = \frac{(f-1) \cdot r_i^2(i)}{f - r_i^2(i)} \sim t_{f-1}
      \]

      Beckman and Trussell (1974); Pope (1976); Heck (1980, 1981)

      Outlier detected at significance level $\alpha$ if $T(i) > f_{1-\alpha/2}$

A linear regression model

- Estimation of regression coefficients
  - Bootstrap estimation (e.g., Efron 1982, chap. 5; Trauth 2006, p. 66 ff)
    - Resampling data with replacement
    - Applying OLS estimation to each resampled data set
    - Parameter estimation with a sampling distribution
    - Time-consuming computation
  - Leave-one-out (LOO) cross validation (e.g., Trauth 2006, p. 77 ff)
    - Temporarily removing the $i$-th data point $(x_i, y_i)$
    - Performing OLS regression using the remaining $n-1$ data points
    - Predicting the $i$-th data point based on the regression model $f(x_i)$
    - Computing the $i$-th discrepancy between prediction and observation
    - Calculating the mean error over all $n$ data points (Allen 1974)

\[
\sigma_{LOO} = \left[ \frac{1}{n} \sum_{i=1}^{n} (y_i - f_i(x_i))^2 \right]^{1/2}
\]
Outlier detection

Fig. 3: Example of outlier detection and its impact on parameter estimation

Regression coefficients

Fig. 4: Comparison of regression coefficients using different approaches for parameter estimation (without outliers)

- Consistent regression coefficients using different methods
- Reliable results also provided by direct OLS estimation (OLSE)
- Key benefit of OLSE: fast computation
Model error (ME)

- ME: standard deviation of OLS residuals
- ME < 5 mm in most instances
- Improved ME after outlier elimination
- Larger ME values on wet days

![Graph showing model error (ME)](image)

Fig. 5: Influences of precipitation on residual standard deviation (parameter estimation: OLSE)

Model verification

![Graphs showing model verification for GPS sites with meteorological data](image)

Fig. 6: Model validation using GPS sites with meteorological data
Model accuracy (MA)

**MA: mean absolute deviation (MAD)**

\[
\text{MAD} = \frac{1}{n} \sum_{i=1}^{n} |ZDD(MET_i) - ZDD_{\text{corrected}}(H_i) - \Delta ZDD_H(i)|
\]

- \(H_i\): altitude of GPS site with meteorological data \((MET_i)\)

**Tab. 2: Accuracy assessment using GPS sites with meteorological data**

<table>
<thead>
<tr>
<th>GPS site with MET data</th>
<th>dill</th>
<th>efbg</th>
<th>muej</th>
<th>bfo1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude above MSL [m]</td>
<td>181</td>
<td>355</td>
<td>548</td>
<td>647</td>
</tr>
<tr>
<td>MAD [mm] ((\Delta ZDD), with outlier)</td>
<td>4.5</td>
<td>8.2</td>
<td>6.6</td>
<td>6.2</td>
</tr>
<tr>
<td>MAD [mm] ((\Delta ZDD), without outlier)</td>
<td>4.6</td>
<td>7.8</td>
<td>5.2</td>
<td>4.5</td>
</tr>
<tr>
<td>Improvement</td>
<td>-2%</td>
<td>5%</td>
<td>21%</td>
<td>27%</td>
</tr>
</tbody>
</table>

- MA values near 5 mm in most cases
- Improved MA after eliminating outliers

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**Conclusions**

- **Problem:** ZDD of GPS signals without near-site MET data
- **Solution:** a correction model for a priori ZDD
  - Using regional free available MET stations of DWD
  - Linear regression between \(H_M\) and \(\Delta ZDD_H\)
  - Parameter estimation: OLSE, BOOT, CROS
    - Outlier detection based on studentised residuals from OLSE
    - Consistent regression coefficients (OLSE: fast computation)
- **Model quality assessments**
  - Model error (ME): standard deviation of OLS residuals
    - Significantly affected by precipitation
    - Mean ME < 5 mm in the presented case study
  - Model accuracy (MA): mean absolute deviation (MAD)
    - Using 4 GPS sites with MET data
    - MA(D\(ZDD_{\text{MET}} - ZDD_{\text{uncorrected}}\))
    - Near 5 mm in most cases
- Considerably improved ME and MA after outlier elimination
Outlook

- **Model validation**
  - Using more GPS sites with MET data
  - Incorporating more regional MET sites
  - MET data with higher temporal resolution

- **Model extension**
  - Considering site location
  - Height-dependent (1 D) → location-dependent (3 D)

Questions & comments

Thank you very much for your attention!