Ray-traced tropospheric slant delays for space geodetic techniques

Vahab Nafisi
Dudy D. Wijaya
Johannes Böhm
Harald Schuh

Geodätische Woche 2010
Köln, 5-7 Oktober 2010
0. Introduction

• Troposphere delay as an error source for all space geodetic techniques

• Model of total delay as a function of surface meteorological parameters only

• Most commonly used model is that of Saastamoinen (1972)

• Parameter estimation techniques (using the least squares method)

• Mapping function to calculate delays for each observation
Zenith delay can be determined as a function of:

- Meteorological data at the site
- Coordinates (height and latitude of the site)
0. Introduction

• “Direct ray-tracing”, to find slant delay directly, along the true path

• Solution: Suitable mathematical model + Atmospheric data set
  
  2D or 3D
  Numerical Weather models (NWM)
1. Mathematical model for 3D and 2D Ray-tracing systems

- **Eikonal equation**

\[ [\nabla L(\vec{r})]^2 = n(\vec{r})^2 \]

- **Refraction index, as a function of meteorological parameters**

\[ N = k_1 \frac{p_d}{T} + k_2 \frac{e}{T} + k_3 \frac{e}{T^2} \]

\[ n = 1 + N \times 10^{-6} \]
1. Mathematical model for 3D and 2D Ray-tracing systems

- Partial derivatives of the refractivity in spherical coordinate system

- 16 points around a specific point (point of ray-path), for computing partial derivatives $\frac{\partial n}{\partial \lambda}$, $\frac{\partial n}{\partial \theta}$ and $\frac{\partial n}{\partial r}$

- Spline interpolation for value of refractivity (or refractive index) in each point

- Special case: 2D Ray-tracer (ignoring out-of-plane components)

\[
\frac{\partial n}{\partial \theta} = 0 \quad \frac{\partial n}{\partial \lambda} = 0
\]
2. Practical considerations for developing a ray-tracing method at TU Wien

- Linear interpolation for temperature and exponential for pressure and water vapor pressure.

- A pre-calculation process for compiling and converting ECMWF file to refractivity (N) profiles for each grid point.

- Rüeger “best average” constants for refractivity (Rüeger, 2002).

- US standard atmosphere of 1976 for meteorological data above upper limit of ECMWF up to 76 Km.

- Horizontal interpolation (spline, bilinear, weighted mean,...)
3. Results for VIE ray-tracers

Computed delays for TSUKUBA site, using 2D and 3D ray-tracers

5-deg. Elevation angle

7-deg. Elevation angle

10-deg. Elevation angle

15-deg. Elevation angle

red: 2D, green: 3D
3. Results for VIE ray-tracers

Computed delays for TSUKUBA site, using 2D and 3D ray-tracers

20-deg. Elevation angle

30-deg. Elevation angle

50-deg. Elevation angle

70-deg. Elevation angle

red: 2D, green: 3D
3. Results for VIE ray-tracers

Differences in total delay
VIE-2D vs VIE-3D
TSUKUBA 12.08.2008

Elevation Angle (degrees)

Azimuth (degrees)

Difference in delay (mm)
4. Results of ray-tracing campaign

- **Aim**: investigations about effect of different elements of ray-tracers on the final results and preparing benchmarking results for other ray-tracers

- **Task**: Computing total ray-traced delays at every degree (outgoing) elevation (>5°) and azimuth, plus the bending effect at the first epoch (0 UT)

- **Under the umbrella of IAG WG 4.3.3 chaired by Thomas Hobiger**

- **First half of 2010**
Participants in this benchmarking campaign

1) UNB Raytracer: Felipe Nievinski, Landon Urquhart and Marcelo Santos (University of New Brunswick, Canada)

2) KARAT Raytracer (2D and 3D): Thomas Hobiger et al. (NICT, Japan)

3) Horizon-Eikonal: Pascal Gegout (GRGS, Toulouse, France)

4) GFZ: Florian Zus et al. (GFZ, Potsdam, Germany)

5) Vienna Raytracer (2D and 3D): Vahab Nafisi, Dudy Wijaya, Johannes Böhm (Vienna University of Technology, Austria)
### 4. Results of ray-tracing campaign

**Different considerations, in different ray-traces**

<table>
<thead>
<tr>
<th>Ray-tracer</th>
<th>NWM</th>
<th>Upper limit of Trop. (km)</th>
<th>Supplementary atmosphere</th>
<th>Radius of curvature of the Earth</th>
<th>Interpolation methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFZ</td>
<td>ECMWF-model level</td>
<td>150</td>
<td>-</td>
<td>local radius of curvature of the reference ellipsoid</td>
<td>Refractivity: log-linear interpolation</td>
</tr>
<tr>
<td>Horizon-eikonal</td>
<td>ECMWF-model level</td>
<td>80</td>
<td>-</td>
<td>WGS84 reference ellipsoid</td>
<td>Refractivity: exponential</td>
</tr>
<tr>
<td>KARAT-Thayer</td>
<td>ECMWF-pressure level</td>
<td>86</td>
<td>US 76</td>
<td>Euler’s formula</td>
<td>Temperature and relative humidity: linear</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Pressure: logarithmically</td>
</tr>
<tr>
<td>KARAT-Eikonal</td>
<td>ECMWF-pressure level</td>
<td>86</td>
<td>US 76</td>
<td>Euler’s formula</td>
<td>Temperature and relative humidity: linear</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Pressure: logarithmically</td>
</tr>
<tr>
<td>UNB-bent3D</td>
<td>ECMWF-pressure level</td>
<td>100</td>
<td>CIRA86</td>
<td>ellipsoidal coordinates</td>
<td>Temperature and specific humidity: linear, Pressure: logarithmically</td>
</tr>
<tr>
<td>VIE-3D</td>
<td>ECMWF-pressure level</td>
<td>76</td>
<td>US 76</td>
<td>Gaussian mean curvature</td>
<td>Temperature: linear</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Pressure and water vapor pressure: exponential</td>
</tr>
<tr>
<td>VIE-2D</td>
<td>ECMWF-pressure level</td>
<td>76</td>
<td>US 76</td>
<td>Gaussian mean curvature</td>
<td>Temperature: linear</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Pressure and water vapor pressure: exponential</td>
</tr>
</tbody>
</table>
4. Results of ray-tracing campaign

<table>
<thead>
<tr>
<th>Data set # 1</th>
<th>Data set # 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSUKUBA site (Japan)</td>
<td>WETTZELL site (Germany)</td>
</tr>
<tr>
<td>$\phi = 36.1031^\circ$</td>
<td>$\phi = 49.15^\circ$</td>
</tr>
<tr>
<td>$\lambda = 140.0887^\circ$</td>
<td>$\lambda = 12.88^\circ$</td>
</tr>
<tr>
<td>$h = 85.09m$</td>
<td>$h = 699.56m$</td>
</tr>
<tr>
<td>$N = 38.92m$</td>
<td>$N = 50m$</td>
</tr>
<tr>
<td>Epoch: 12.08.2008, $0^h$</td>
<td>Epoch: 01.01.2008, $0^h$</td>
</tr>
<tr>
<td>ECMWF Model, 0.1×0.1</td>
<td>ECMWF Model, 0.1×0.1</td>
</tr>
</tbody>
</table>
4. Results of ray-tracing campaign – Tsukuba, 5 deg. Elevation angle (5)
4. Results of ray-tracing campaign – Tsukuba, 5 deg. Elevation angle (6)
4. Results of ray-tracing campaign – Wettzell, 5 deg. Elevation angle (7)

01.01.2008
4. Results of ray-tracing campaign – Wettzell, 5 deg. Elevation angle (8)
4. Results of ray-tracing campaign – remarks

**Rule of thumb:** The error in the station height is approximately 1/5 of the slant delay error at the 5 degrees elevation angle (Böhm, 2004)

For precision of 2mm for station heights we must care about elements of ray-tracer system
For a precision of 2 mm in station height, the biases between slant factors have to be smaller than 0.005. If the zenith delay is 2 m this corresponds to 1 cm at 5 degrees elevation, and 1/5th of this is 2 mm.

We need to estimate residual zenith delay
5. Concluding remarks and future works

- Developing a 3D ray-tracer based on Eikonal equation
- Some simplifications for a 2D ray-tracer (ignoring out-of-plane components)
- Discrepancies between results of different ray-tracers (ray-tracing campaign), because of different interpolation and extrapolation methods, upper limit of troposphere, data sets
5. Concluding remarks and future works

- Focus on different Numerical Weather Models in future
- Validation of results obtained by VIE ray-tracers, using VLBI observations
- Ray-traced delays as a part of Vienna VLBI Software (VieVS) in future