Canonical Correlation Analysis (CCA) of GRACE, hydrological and hydro-meteorological signals

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Geodetic Week, Oct. 2010
\[ P - ET_a - R = \frac{dS_H}{dt} \iff \frac{dM}{dt} \iff \frac{dS_A}{dt} = -\nabla \cdot Q - R \]

Hydrology

GRACE

Hydro-meteorology
Residual contains:

- Error
- Physical variations
EOF analysis

\[ A = U \Sigma V^T \]

- \( U \): Eigenvectors of matrix \( AA^T \)
- \( \Sigma \): Eigenvalues of matrix \( A^T A \) or \( AA^T \)
- \( V \): Eigenvectors of matrix \( A^T A \)
EOF analysis

\[
\frac{dM}{dt} = U_G \Sigma_G V_G^T
\]

GRACE

\[
P = U_p \Sigma_p V_p^T
\]

Precipitation

\[
\nabla \cdot Q = U_D \Sigma_D V_D^T
\]

Vertically integrated moisture flux divergence
Canonical Correlation Analysis (CCA)

\[ A = U_A \Sigma_A V_A^T \]
\[ B = U_B \Sigma_B V_B^T \]

\[ A^T B = U_{AB} \Sigma_{AB} V_{AB}^T \]

\[ \Sigma_{AB} = U_A^T A_{AB} U_B = K(U_A^T U_B) \]

- Measuring the linear relationship between two multi dimensional variables
- Finding two sets of basis vectors such that the correlation between the projections of the variables onto these basis vectors is maximized
- Determine Correlation Coefficients
- Correlation coefficients: Proportion of correlation between the canonical variates accounted for the particular variable
- Correlation coefficient represents unique contribution of each variable to relation
CCA of GRACE \( \left( \frac{dM}{dt} \right) \) and Divergence \( (\nabla \cdot Q) \)

\[
\left( \frac{dM}{dt} \right)^T \nabla \cdot Q = U_{GD} \Sigma_{GD} V_{GD}^T
\]
CCA of GRACE \( \left( \frac{dM}{dt} \right) \) and Divergence \( \vec{\nabla} \cdot \vec{Q} \)

\[
\left( \frac{dM}{dt} \right)^T \vec{\nabla} \cdot \vec{Q} = U_{GD} \Sigma_{GD} V_{GD}^T
\]

Mode 3

Correlation coefficient : 88%

Projection of \( U_{GD} \)

Projection of \( V_{GD} \)

Mode 1

Correlation coefficient : 38%

Mode 2

Correlation coefficient : 11%

Mode 3

Correlation coefficient : 8%
CCA of GRACE \( \left( \frac{dM}{dt} \right) \) and Precipitation \( (P) \)

\[
\begin{align*}
\left( \frac{dM}{dt} \right)^T P &= U_{GP} \Sigma_{GP} V_{GP}^T
\end{align*}
\]
CCA of GRACE $\left(\frac{dM}{dt}\right)$ and Precipitation ($P$)

$\left(\frac{dM}{dt}\right)^T P = U_{GP} \Sigma_{GP} V_{GP}^T$

Mode 3

Correlation coefficient : 20%
CCA on catchments based signals
CCA on catchments based – GRACE and hydro-meteorology

\[
\left( \frac{dM}{dt} \right)^T \nabla \cdot Q = U_{GD} \Sigma_{GD} V_{GD}^T
\]
CCA on catchments based – GRACE and hydro-meteorology

\[
\frac{dM}{dt} \leftrightarrow -\nabla \cdot Q - R \quad \text{Residual}
\]
CCA on catchments based – GRACE and precipitation

\[
\left( \frac{dM}{dt} \right)^T P = U_{GP} \Sigma_{GP} V_{GP}^T
\]
CCA on catchments based – GRACE and Precipitation

\[
\frac{dM}{dt} \leftrightarrow P - R \quad \text{Residual}
\]
Summary and Outlook

Summary

• CCA was performed on GRACE, hydrological and hydro-meteorological signals
• Selecting the 75% of correlation of GRACE, hydrological and hydro-meteorological signals leads to:
  • Improvement in correlation of signals
  • Decreasing the RMS of residual
• The correlation of GRACE, hydrological and hydro-meteorological residuals does not show improvement or deterioration

Outlook

• Applying CCA on different climate regions
• Using other source of data sets like temperature
Thanks for your attention