A residual decomposition model for determining and modelling temporal correlations of GPS observations

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Introduction

- **Temporal correlations of GPS observations**
  - Usually neglected in the stochastic model
  - Modelling approaches based on residual time series
    (e.g., Wang et al. 2002; Tiberius and Kenselaar 2003; Howind 2005)

- **Residuals from GPS data processing**
  - Systematic components
    - Deficiencies in the functional model
    - Remaining atmospheric effects
    - Site-specific influences (e.g., multipath effects)
  - Noise component
    - Deficiencies in the stochastic model
    - E.g., temporal correlations of GPS observations

- **Objective: residual-based analyses of temporal correlations**
  - Separating noise from the remaining systematic effects
  - Residual decomposition
Residual decomposition model

- **Studentised residuals** (Cook and Weisberg 1982, p. 18)
  \[ r_i = \frac{v_i}{\sigma_v} \]

  Pope (1976); Heck (1981); Howind (2005, p. 39)

  - More homoscedastic than least-squares residuals \( v \)
  - Containing information on temporal correlations

- **Decomposition model** (e.g., Brockwell and Davis 2002, p. 23, 31)
  \[ Y_i = m_i + s_i + X_i \]

  \( m_i \): slowly changing trend component → Vondrák filter
  \( s_i \): quasi-periodic component → Temporal stacking
  \( X_i \): weakly stationary noise component → ARMA modelling

ARMA: autoregressive moving average

Vondrák filter (Whittaker and Robinson 1946; Vondrák 1969, 1977)

- **Absolute fitting**: \( F = 0 \)
  \[ F = \sum_{i=1}^{n} p_i (y_i' - y_i)^2 \]

  \( y_i / y_i' \): observed / filtered value at \( x_i \)
  \( p_i \): weight of observation \( y_i \)

- **Absolute smoothing**: \( S = 0 \)
  \[ S = \int_{-\infty}^{\infty} |\varphi'''(x)|^2 dx \]

  \( \varphi'''(x) \): 3rd derivative of \( \varphi(x) = y'(x) \)

- **A compromise between absolute fitting and smoothing**
  \[ Q = F + \lambda^2 S \rightarrow \min \Rightarrow \partial Q / \partial y_i' = 0 \rightarrow y_i' \]

  - \( \lambda = 0 \rightarrow F = 0 \rightarrow y_i' = y_i \): absolute fitting
  - \( \lambda \rightarrow \infty \rightarrow S = 0 \wedge F \rightarrow \min \) least-squares quadratic polynomial fitting
  - Unknown \( \varphi(x) \) → using a 3rd-order Lagrange polynomial \( \varphi(x) = L_3(x) \)
**Vondrák filter**

![Image of Vondrák filter example](image)

**Fig. 1:** Examples when applying Vondrák filter with different smoothing factors.

- No pre-defined functions required
- Filter values at the two ends of the series available
- Applicable for equidistant and non-equidistant data
- Performance not degraded by non-linearity

(Vondrák 1969; Brockwell and Davis 2002, p. 27; Zheng et al. 2005)

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**Temporal stacking** *(e.g., Howind 2005, p. 55)*

- **Basic assumptions**
  - Repeating GPS satellite geometry
  - Unchanged environment of observation station

- **Geometry-repeat lag**
  - Nominal value: 86164 s (23 h 56 m 4 s, mean sidereal day)
  - Optimum value: 86154 s (Choi et al. 2004; Ragheb et al. 2007)

![Image of Temporal stacking example](image)

**Fig. 2:** Temporal stacking by calculating epoch-wise mean values.

\[
\hat{y}_i(t) = \frac{1}{N} \sum_{j=1}^{N} (y^j_{i} - \bar{y}_i) \quad i = 1, \ldots, n
\]
ARMA modelling

- **ARMA process** (e.g., Broersen 2006, p. 74)
  \[ X_t + a_1 X_{t-1} + \cdots + a_p X_{t-p} = Z_t + b_1 Z_{t-1} + \cdots + b_q Z_{t-q} + \epsilon_t \]
  \[ (p, q): \text{order parameters} \]
  \[ (a_1, \ldots, a_p, b_1, \ldots, b_q): \text{ARMA coefficients} \]
  \[ \sigma^2: \text{white noise (WN) variance} \]

- **Model identification and determination**
  - Free software: ARMASA toolbox (MATLAB Central)
  - Order selection
    - AR model: Combined Information Criterion (Broersen 2000a)
    - MA and ARMA model: Generalised Information Criterion (Broersen 2000b)
  - Parameter estimation
    - AR model: Burg’s method (Burg 1967)
    - MA and ARMA model: Durbin’s 1st and 2nd methods (Durbin 1959, 1960)
  - Model selection: minimum prediction error (Broersen 2006, p. 99)

Statistical hypothesis tests

- **Unit root tests for stationarity**
  - Augmented Dickey-Fuller (ADF) test (Said and Dickey 1984)
    - H0: non-stationary / H1: (trend-)stationary
  - Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test (Kwiatkowski et al. 1992)
    - H0: (trend-)stationary / H1: non-stationary
  - Linear trend model (impact on critical values)

- **Tests for uncorrelatedness**
  - Based on empirical autocorrelation function (ACF)
    - Von Neumann ratio (VNR) (Bingham and Nelson 1981)
    - Ljung-Box (LB) portmanteau test (Ljung and Box 1978)
  - Based on empirical power spectral density (PSD)
    - Kolmogorov-Smirnov (KS) test (Teusch 2006, p. 103)
    - Cramér-von Mises (CM) test (Teusch 2006, p. 104)

- **Significance level for all tests:** \( \alpha = 5\% \)
  - Probability of type I error (wrongly rejecting H0)
Residual data set

Tab. 1: GPS data processing strategy (Fuhrmann et al. 2010)

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<th>Processing technique</th>
<th>Precise Point Positioning (PPP) Bernese GPS Software 5.0</th>
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<tr>
<td>Stations (MP: multipath)</td>
<td>Tübingen (TUEB, MP: weak) Bingen (BING, MP: strong)</td>
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<td>Observations DOY08:275-284</td>
<td>30 s GPS phase observations Ionosphere-free linear combination (L3)</td>
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<td>Obs. weighting</td>
<td>sin^2 ε (elevation cut-off: 10°)</td>
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<td>Orbits, EOP¹, satellite clocks</td>
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<td>Tropospheric models</td>
<td>Saastamoinen + Niell (MF) Troposphere parameters (30 min) Gradient parameters (24 h)</td>
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<td>Antenna models</td>
<td>IGS standard 2006 (satellite) Individual absolute (receiver)</td>
</tr>
</tbody>
</table>

EOP: Earth Orientation Parameters

516 time series of studentised PPP residuals with \( n \geq 200 \) epochs (100 min)

Residual times series

Fig. 3: Exemplary illustration of retrieved remaining systematic effects (PRN11, DOY2008:275-284)
Sample ACF

- Correlation length based on the first zero point of sample ACF
- Determined correlation lengths: 3-15 min
- Increasing multipath impact → decreasing correlation length
- Increasing satellite elevation angle → increasing correlation length

Tests for stationarity

- Widely consistent test results using opposite H0
- Non-stationarity of RAW primarily due to long-periodic trends
- Efficiency of Vondrák filter in modelling trend component
- Stationary STK → pre-condition for ARMA modelling
Tests for uncorrelatedness

Largely consistent results using different tests
Statistically significant temporal correlations in STK → incorporation into the stochastic model of GPS data processing
Efficiency of ARMA processes in modelling temporal correlations

Conclusions

Residual-based correlation analyses of GPS observations
- Separating noise from the remaining systematic effects
  - Vondrák filter → long-periodic trends
  - Temporal stacking → quasiperiodic signals induced by MP
- Modelling temporal correlations by means of ARMA processes

A case study using PPP residual time series
- Temporal correlation lengths: 3-15 min
  - Based on sample ACF of the decomposed noise
  - Dependencies on multipath and satellite elevation
- Statistical tests for stationarity and uncorrelatedness
  - Non-stationarity mainly due to long-periodic trends
  - Decomposed noise: stationary, significantly correlated in time
  - Efficiency of residual decomposition and ARMA modelling
Outlook

- **Physical causes of long-periodic trends**
  - Remaining atmospheric effects
  - Antenna near-field influences

- **Improving the mathematical models for GPS observations**
  - Extracted systematic effects → functional model
  - ARMA-modelled temporal correlations → stochastic model

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**Fig. 7:** VCM for zero-difference L3 observations (BING, DOY2008:275, 20 epochs: 600 s)

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Questions & comments

Thank you very much for your attention!

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