Local Gravity Field Modeling by 2DFFT from Gravity Gradients

Wei Zhao, Nico Sneeuw

Geodätisches Institut der Universität Stuttgart

[zhao,sneeuw]@gis.uni-stuttgart.de

1. Introduction

THE gravity disturbing potential T can be calculated by gravity gradient components $T_{ij}(T_{xx}, T_{xy}, T_{xz}, T_{yy}, T_{yz}, T_{zz})$ separately. First calculate the disturbing potential by each FTG components, then the consistency of the results are compared. For the representation of regional gravity data, Fourier Series representation which can express the solution of the Laplace equation in cartesian coordinates is used.

2. Theory

The solution of the Laplace equation ($\Delta\phi(x, y, z) = 0$ for $z > 0$),

$$\Phi(x, y, z) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(a_{nm}\cos nx\cos my + b_{nm}\cos nx\sin my + c_{nm}\sin nx\cos my + d_{nm}\sin nx\sin my)e^{-\sqrt{n^2+m^2}z}$$

Here, $\Phi(x, y, z)$ satisfies the regularity condition. It can be disturbing potential T, gravity anomaly Δg, or gravity gradient components T_{ij}. If we represent disturbing potential T as,

$$T = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(a_{nm}\cos nx\cos my + b_{nm}\cos nx\sin my + c_{nm}\sin nx\cos my + d_{nm}\sin nx\sin my)e^{-\sqrt{n^2+m^2}z}$$

and gravity gradient components T_{ij} as,

$$T_{ij} = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(p^{ij}_{nm}\cos nx\cos my + q^{ij}_{nm}\cos nx\sin my + r^{ij}_{nm}\sin nx\cos my + s^{ij}_{nm}\sin nx\sin my)e^{-\sqrt{n^2+m^2}z}$$

Because gravity gradient components T_{ij} are the second derivatives of disturbing potential T in spatial domain,

$$T_{ij} = \frac{\partial^2 T}{\partial x_i \partial x_j}$$

the relationship between T and T_{ij} can be calculated in spectral domain (see Table1),

Table 1: *Pocket Guide*

	p^{ij}_{nm}	q^{ij}_{nm}	r^{ij}_{nm}	s^{ij}_{nm}
T_{xx}	$-n^2 a_{nm}$	$-n^2 b_{nm}$	$-n^2 c_{nm}$	$-n^2 d_{nm}$
T_{xy}	nmd_{nm}	$-nmc_{nm}$	$-nmb_{nm}$	nma_{nm}
T_{xz}	$-n\sqrt{n^2+m^2}c_{nm}$	$-n\sqrt{n^2+m^2}d_{nm}$	$n\sqrt{n^2+m^2}a_{nm}$	$n\sqrt{n^2+m^2}b_{nm}$
T_{yy}	$-m^2 a_{nm}$	$-m^2 b_{nm}$	$-m^2 c_{nm}$	$-m^2 d_{nm}$
T_{yz}	$-m\sqrt{n^2+m^2}b_{nm}$	$m\sqrt{n^2+m^2}a_{nm}$	$-m\sqrt{n^2+m^2}d_{nm}$	$m\sqrt{n^2+m^2}c_{nm}$
T_{zz}	$(n^2+m^2)a_{nm}$	$(n^2+m^2)b_{nm}$	$(n^2+m^2)c_{nm}$	$(n^2+m^2)d_{nm}$

In the numerical calculation procedure, first interpolate the original data to get a grid data and then calculate the spectra of the interpolated data for each gravity gradient component. Using the pocket guide to calculate the spectra of disturbing potential T, finally we obtain the disturbing potential in spatial domain.

3. Data

THE dataset used here which contains all FTG Tensor Components was measured by Bell Geospace Ltd. The data was measured by ship with a gradiometer in a nearly $60\,\mathrm{km} \times 80\,\mathrm{km}$ ocean surface which is between Faroe Islands and Shetland Islands. A approximate $15\,\mathrm{m}$ distance between the sampling points is estimated roughly. Centimeter precise ED50 coordinates of the sampling points are given and transferred to UTM coordinates (the UTM coordinates are used in the following interpolation procedure).

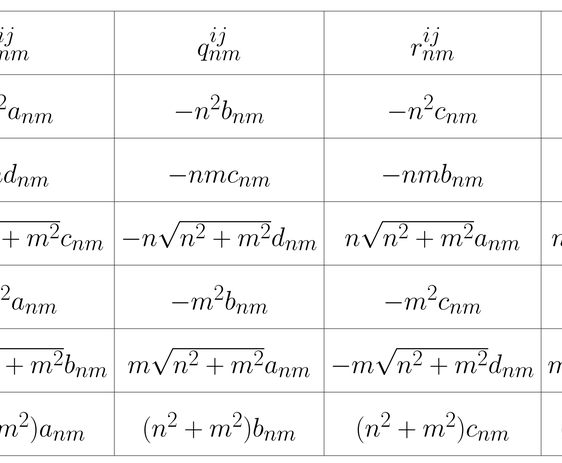

Figure 1: *Trajectories of the Sampling Points, data in the red rectangle is used.*

4. Results

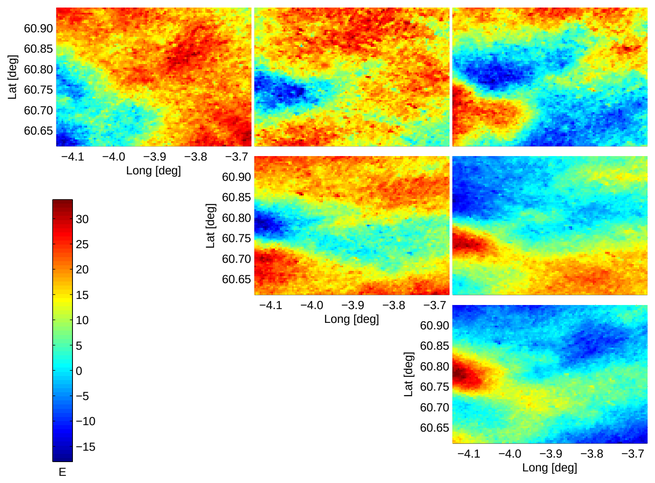

Figure 2: *Interpolated FTG Components*

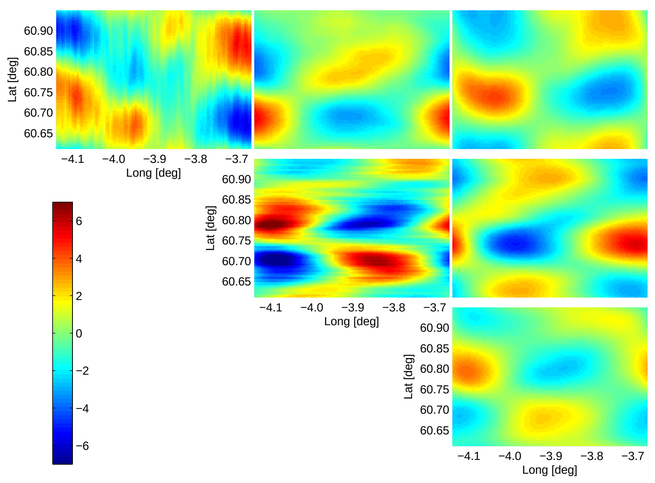

Figure 3: *Disturbing Potential Calculated by FTG Components*

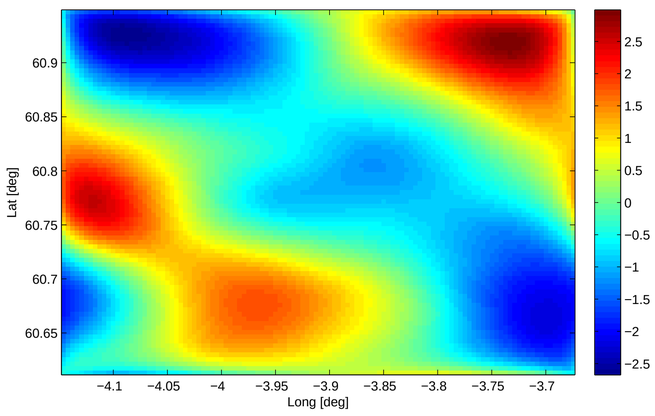

Figure 4: *Disturbing Potential Calculated by Gravity Anomaly*

5. Discussion

- The transformation procedure of the spectra between T_{ij} and T ignored the low wave number terms whenever $n = 0$ or $m = 0$.
- The stripes in Figure 3 which only happened in the results that calculated by T_{xx} and T_{yy} components looks strange, if we see the pocket guide in Table 1 we can see that for T_{xx} and T_{yy} components they only related to wave number n and m separately. This may explain why the stripes existing here.
- The consistency of disturbing potential T calculated by each gravity gradient components T_{ij} are supposed to be significantly, but from Figure 3 it can not be seen clearly. Some more details should be discussed in future work.
- Comparing T calculated by gravity anomaly Δg with T computed by T_{ij}, the result of T_{zz} has more consistency with the result of T_{ze}.

References

[1] N. Sneeuw, *Physical Geodesy*, (2006), Lecture Notes, University of Stuttgart.

[2] R. Rummel, *Satellite Gradiometry,* (1986), Mathematical and Numerical Techniques in Physical Geodesy, Springer, 7:317-363

[3] R. Rummel, M. van Gelderen, *Spectral analysis of the full gravity tensor,* (1992), Geophys. J. Int, 111:159-169