Gravity field determination from GOCE using the time-wise method – status of release 3

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Nürnberg, 29.09.2011
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   - Input details
   - Comparing the releases in terms of coefficients
   - Comparing the releases in terms of estimated standard deviations
   - Comparing the releases in terms of gravity anomalies
   - Comparing the releases in terms of propagated errors

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4 Summary and Outlook
Sensors for gravity field determination

The whole satellite is the sensor!

But three main sensor observations to be scientifically processed for gravity field determination:

- GPS tracking (SST)
- Gradiometry (SGG)
- Star tracker (STR)
From observations to the gravity field

\[ V(r, \theta, \lambda) = \frac{GM}{a} \sum_{l=0}^{l_{\text{max}}} \left( \frac{a}{r} \right)^{l+1} \sum_{m=0}^{l} (c_{lm} \cos(m\lambda) + s_{lm} \sin(m\lambda)) P_{lm}(\cos\theta), \ (1) \]

GOCE-only gravity field model independent of any other gravity field information

spherical harmonic coefficients
+ accuracies as full VCM
geoid heights
+ accuracies
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4 Summary and Outlook
Time-Wise Approach

gap-less part of independent gradients as equidistant time series:

Observation equations:
1. potential in EFRF cf. (1)
2. 2\textsuperscript{nd} derivative according to \(\lambda, \theta\)
3. rotation to GRF (earth rotation, error-free STR) \((1 + \mathbf{v} = \mathbf{Ax})_{\text{GRF}}\)
4. application of decorrelation filters to obs. eqs. \((\mathbf{F}\mathbf{l} + \mathbf{F}\mathbf{v} = \mathbf{F}\mathbf{Ax})_{\text{GRF}}\)
5. Final obs. eqs.: \((\mathbf{I} + \mathbf{v} = \mathbf{Ax})_{\text{GRF}}', Q_{\II} = \mathbf{I}\)

Solution process Tuning machine:
1. Least-Squares solution, using fast parallel iterative solver (PCGMA, conjugate gradients)
2. combination: SGG OEQ + SST NEQ
3. stabilisation using Kaula’s rule
4. optimal relative weighting using VCE
5. iterative refinement of filter cascades
6. iterative outlier detection based on decorrelated gradient residuals

Results as input for Coresolver:
- gravity field solution + approx. VCM
- stochastic model: converged filters + weights for SST, SGG and REG
- quality information: outlier flags

Solution process Coresolver:
- Assembling of full NEQ
- Solution of full NEQ
- Inversion of full NEQ
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4 Summary and Outlook
The three releases, some details

<table>
<thead>
<tr>
<th>Release</th>
<th>Dates</th>
<th>SST Observations</th>
<th>SST Flags</th>
<th>SGG Observations</th>
<th>SGG Flags</th>
<th>Segments</th>
</tr>
</thead>
<tbody>
<tr>
<td>EGM_TIM_RL1</td>
<td>23-Nov-2009 - 11-Jun-2010</td>
<td>3× 6 161 897</td>
<td>3× 148 104</td>
<td>3× 6 161 834</td>
<td>3× 10 905 (0.2%) + $V_{yy}$ SA</td>
<td>1</td>
</tr>
<tr>
<td>EGM_TIM_RL2</td>
<td>03-Mar-2010 - 19-Sep-2010</td>
<td>3× 21 427 200</td>
<td>3× 1 896 934</td>
<td>3× 19 477 946</td>
<td>3× 559 573 (2.9%) + $V_{yy}$ SA</td>
<td>8</td>
</tr>
<tr>
<td>EGM_TIM_RL3p</td>
<td>11-Jun-2010 - 28-Dec-2010</td>
<td>3× 31 289 605</td>
<td>3× 460 682 (1.5%)</td>
<td>3× 31 289 605</td>
<td>3× 460 682 (1.5%)</td>
<td>16</td>
</tr>
</tbody>
</table>
## Combined model

\[
(\omega_{sst} N_{sst} + \omega_{sgg} A_{sgg}^T F_{sgg}^T F_{sgg} A_{sgg} + \omega_{reg} N_{reg}) \mathbf{x} = \omega_{sst} n_{sst} + \omega_{sgg} A_{sgg}^T F_{sgg}^T l_{sgg} + \omega_{reg} n_{reg}
\]

\[
\mathbf{x} \equiv \mathbf{n}
\]

<table>
<thead>
<tr>
<th>SST</th>
<th>method</th>
<th>stochastic model</th>
<th>weight</th>
<th>resolution</th>
<th>basis</th>
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</thead>
<tbody>
<tr>
<td>RL1</td>
<td>energy balance</td>
<td>propagated PCV</td>
<td>VCE</td>
<td>d/o 2-100</td>
<td>full NEQ</td>
</tr>
<tr>
<td>RL2</td>
<td>energy balance</td>
<td>propagated PCV</td>
<td>VCE</td>
<td>d/o 2-100</td>
<td>full NEQ</td>
</tr>
<tr>
<td>RL3p</td>
<td>energy balance</td>
<td>propagated PCV</td>
<td>VCE</td>
<td>d/o 2-100</td>
<td>full NEQ</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>SGG</th>
<th>method</th>
<th>stochastic model</th>
<th>weight</th>
<th>resolution</th>
<th>basis</th>
<th>(V_{yy})</th>
<th>SA</th>
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</thead>
<tbody>
<tr>
<td>RL1</td>
<td>time-wise</td>
<td>ARMA filter</td>
<td>VCE</td>
<td>d/o 2-224</td>
<td>full NEQ</td>
<td>flagged</td>
<td></td>
</tr>
<tr>
<td>RL2</td>
<td>time-wise</td>
<td>ARMA filter/segment</td>
<td>VCE</td>
<td>d/o 2-250</td>
<td>full NEQ</td>
<td>flagged</td>
<td></td>
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<tr>
<td>RL3p</td>
<td>time-wise</td>
<td>ARMA filter/segment</td>
<td>VCE</td>
<td>d/o 2-250</td>
<td>iter. TM</td>
<td>–</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>REG</th>
<th>method</th>
<th>stoch. model</th>
<th>weight</th>
<th>resolution</th>
<th>basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>RL1</td>
<td>(c, s_{lm} = 0)</td>
<td>Kaula</td>
<td>1 VC, VCE</td>
<td>near zonals+d/o170-224</td>
<td>full NEQ</td>
</tr>
<tr>
<td>RL2</td>
<td>(c, s_{lm} = 0)</td>
<td>Kaula</td>
<td>2 VC, VCE</td>
<td>near zonals+d/o180-250</td>
<td>full NEQ</td>
</tr>
<tr>
<td>RL3p</td>
<td>(c, s_{lm} = 0)</td>
<td>Kaula</td>
<td>2 VC, VCE</td>
<td>near zonals+d/o180-250</td>
<td>full NEQ</td>
</tr>
</tbody>
</table>

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GOCE Gravity Field Model
GOCE-only model performance

GOCE performance compared to ITG-Grace2010s:

solid: degree error variance from difference to ITG-Grace2010s, dashed: degree error variance from formal errors
GOCE-only model performance

GOCE performance compared to ITG-Grace2010s:

- **solid**: degree error variance from difference to ITG-Grace2010s,
- **dashed**: degree error variance from formal errors

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GOCE Gravity Field Model
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GOCE performance compared to ITG-Grace2010s:

**solid:** degree error variance from difference to ITG-Grace2010s, **dashed:** degree error variance from formal errors

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GOCE-only model performance

GOCE performance compared to EGM08:

solid: degree error variance from difference to EGM08, dashed: degree error variance from formal errors
GOCE-only model performance

GOCE performance compared to EGM08:

- **solid**: degree error variance from difference to EGM08
- **dashed**: degree error variance from formal errors

![Graph showing GOCE model performance compared to EGM08](image)

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GOCE Gravity Field Model
GOCE-only model performance

GOCE performance compared to EGM08:

10⁻¹
10⁻²
10⁻³
10⁻⁴
degree variance
0 50 100 150 200 250
sh degree
EGM08 EGM_TIM_RL1
EGM_TIM_RL2 EGM_TIM_RL3p

solid: degree error variance from difference to EGM08, dashed: degree error variance from formal errors
Standard deviations of coefficients

RL01

RL02

RL03p
Comparison to GRACE

Anomalies compared to ITG-Grace2010s (d/o 140, m/s²): RL01

<table>
<thead>
<tr>
<th>sector [°]</th>
<th>min [mGal]</th>
<th>max [mGal]</th>
<th>mean [mGal]</th>
<th>rms [mGal]</th>
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<td>±80.0</td>
<td>-3.919</td>
<td>3.839</td>
<td>0.001</td>
<td>0.654</td>
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<td>±90.0</td>
<td>-67.587</td>
<td>51.001</td>
<td>-0.435</td>
<td>5.557</td>
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GOCE Gravity Field Model
Comparison to GRACE

Anomalies compared to ITG-Grace2010s (d/o 140, $m/s^2$): RL02

<table>
<thead>
<tr>
<th>sector $[^\circ]$</th>
<th>min [mGal]</th>
<th>max [mGal]</th>
<th>mean [mGal]</th>
<th>rms [mGal]</th>
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</thead>
<tbody>
<tr>
<td>±80.0</td>
<td>-3.688</td>
<td>3.941</td>
<td>0.001</td>
<td>0.516</td>
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<td>±90.0</td>
<td>-81.558</td>
<td>48.046</td>
<td>-0.365</td>
<td>5.591</td>
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</tbody>
</table>

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Anomalies compared to ITG-Grace2010s (d/o 140, m/s²): RL03p

<table>
<thead>
<tr>
<th>sector [°]</th>
<th>min [mGal]</th>
<th>max [mGal]</th>
<th>mean [mGal]</th>
<th>rms [mGal]</th>
</tr>
</thead>
<tbody>
<tr>
<td>±80.0</td>
<td>-3.366</td>
<td>3.923</td>
<td>0.001</td>
<td>0.490</td>
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<tr>
<td>±90.0</td>
<td>-84.032</td>
<td>51.363</td>
<td>-0.409</td>
<td>5.906</td>
</tr>
</tbody>
</table>
Comparison to EGM08 I

Anomalies compared to EGM2008 (d/o 200, m/s²) on local scale (Germany, high quality terrestrial data in EGM08):

**RL01**

- min : $-11.3$ mgal
- max : $+10.0$ mgal
- rms : $+3.9$ mgal
- $\bar{\sigma}_g$ : $+3.0$ mgal

**RL02**

- min : $-6.1$ mgal
- max : $+4.7$ mgal
- rms : $+1.7$ mgal
- $\bar{\sigma}_g$ : $+1.7$ mgal

**RL03p**

- min : $-5.3$ mgal
- max : $+3.3$ mgal
- rms : $+1.4$ mgal
- $\bar{\sigma}_g$ : $+1.4$ mgal

**GOCE Gravity Field Model**
Comparison to EGM08 II

Anomalies compared to EGM2008 (d/o 200, m/s²) on local scale (South America, low quality terestrial data in EGM08):

**RL01**
- min : $-66.4$ mgal
- max : $+58.0$ mgal
- $\bar{\sigma}_g : +3.6$ mgal

**RL02**
- min : $-64.8$ mgal
- max : $+56.1$ mgal
- $\bar{\sigma}_g : +2.1$ mgal

**RL03p**
- min : $-65.1$ mgal
- max : $+55.4$ mgal
- $\bar{\sigma}_g : +1.7$ mgal

**diff.EGM08**
- min : $-50.0$ mgal
- max : $+40.0$ mgal
- $\bar{\sigma}_g : +0.0$ mgal

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GOCE Gravity Field Model
Comparison to EGM08 III

Anomalies compared to EGM2008 (d/o 200, \( m/s^2 \)) on local scale (North Atlantic, altimetry in EGM08):

**RL01**
- min : \(-10.7\) mgal
- max : \(+11.1\) mgal
- rms : \(+3.1\) mgal
- \( \bar{\sigma}_g : +3.1\) mgal

**RL02**
- min : \(-7.4\) mgal
- max : \(+8.1\) mgal
- rms : \(+2.0\) mgal
- \( \bar{\sigma}_g : +1.7\) mgal

**RL03p**
- min : \(-5.0\) mgal
- max : \(+5.4\) mgal
- rms : \(+1.5\) mgal
- \( \bar{\sigma}_g : +1.4\) mgal

diff. EGM08
var. prop.
Propagated anomaly errors

Propagated gravity anomaly errors (d/o 200, \(m/s^2\)): RL01

<table>
<thead>
<tr>
<th>sector [°]</th>
<th>min [mGal]</th>
<th>max [mGal]</th>
<th>mean [mGal]</th>
</tr>
</thead>
<tbody>
<tr>
<td>±80.0</td>
<td>2.197</td>
<td>4.269</td>
<td>3.322</td>
</tr>
<tr>
<td>±90.0</td>
<td>2.197</td>
<td>28.214</td>
<td>4.594</td>
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</table>

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Propagated anomaly errors

Propagated gravity anomaly errors (d/o 200, \(m/s^2\)): RL02

<table>
<thead>
<tr>
<th>sector</th>
<th>min [mGal]</th>
<th>max [mGal]</th>
<th>mean [mGal]</th>
</tr>
</thead>
<tbody>
<tr>
<td>±80.0</td>
<td>1.405</td>
<td>3.097</td>
<td>1.974</td>
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<tr>
<td>±90.0</td>
<td>1.405</td>
<td>25.473</td>
<td>3.236</td>
</tr>
</tbody>
</table>

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GOCE Gravity Field Model
Propagated gravity anomaly errors (d/o 200, \( m/s^2 \)): RL03p

<table>
<thead>
<tr>
<th>sector</th>
<th>min [mGal]</th>
<th>max [mGal]</th>
<th>mean [mGal]</th>
</tr>
</thead>
<tbody>
<tr>
<td>±80.0</td>
<td>1.178</td>
<td>1.967</td>
<td>1.579</td>
</tr>
<tr>
<td>±90.0</td>
<td>1.178</td>
<td>25.264</td>
<td>2.860</td>
</tr>
</tbody>
</table>

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4 Summary and Outlook
Gain of the current GOCE models:

- consistent set of spherical harmonic coefficients d/o 2-250
- high quality variance/covariance matrix
- GRACE improvement d/o 100+
- EGM08 improvement d/o 60-180
- improvements with the new release as expected
- mean global accuracy at d/o 200: 1.6 mgal, 5.6 cm

Outlook:

- ESA is reprocessing L1b gravity gradients with improved algorithm
- data up to at least end of 2012
- full NEQs are assembling, south australia flagged for ascending tracks $V_{yy}$
- probably including $V_{xz}$
- update of SST normals covering the full period
- update of relative weighting
- official Release of version 3 GOCE models in Nov 2011