Effiziente Umsetzung der Integration der Elektronendichte innerhalb der Ionosphäre entlang des Signalweges

(DFG-Projekt MuSIK)

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Outline

Modeling

Integration

Analysis

Conclusion

- The geometry free linear combination from double frequency carrier phase GNSS observations can be used to eliminate geometry, i.e. satellite orbits, station positions, clock errors and tropospheric delays

$$\Phi(R,S,t)_4 = \Phi_1 - \Phi_2$$
$$= \alpha \cdot STEC(R,S,t) + \beta_R + \beta_S + CPB + \varepsilon$$

with $STEC(R,S,t)$: Slant Total Electron Content

$\beta_S\ \beta_R$: Inter-frequency differential delays

CPB : Carrier phase bias

ε : Noise

- STEC is defined as the integral of the space- and time-dependent electron density Ne along the ray path between satellite S and receiver R

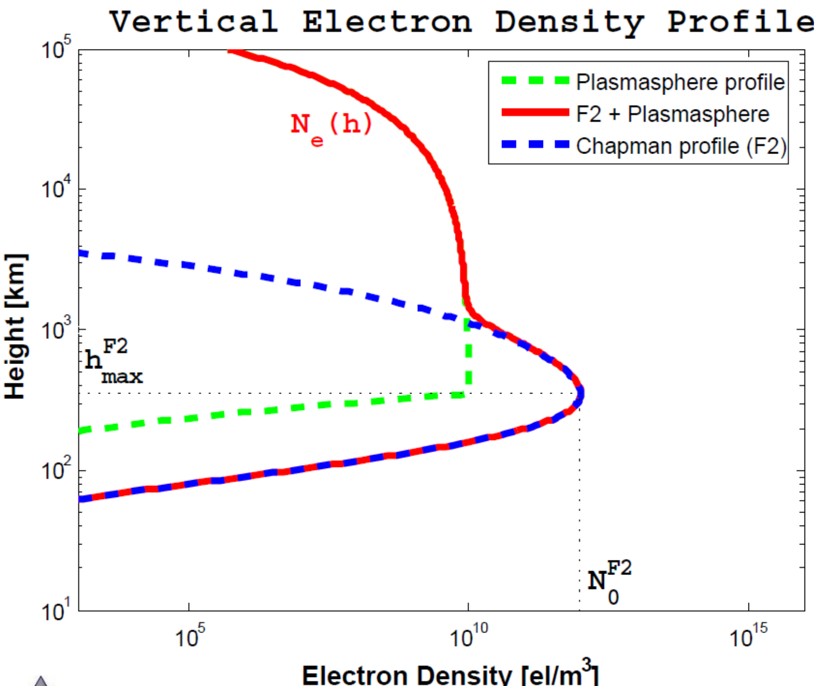
$$\boxed{STEC(R,S,t)_{obs} = \int_R^S Ne(x,t)\,ds}$$

wherein $x = r \cdot \begin{pmatrix} \cos\varphi\cos\lambda \\ \cos\varphi\sin\lambda \\ \sin\varphi \end{pmatrix}$

x : Geocentric position vector
φ, λ : Latitude, longitude
t : Time
$r = \rho + h$: Geocentric distance

- The height dependency of the electron density can be modelled by the physics-motivated Chapman function for the F2-layer in combination with a plasmasphere profile

$$STEC(R,S,t)_{obs} = \int_R^S \left(\underbrace{N_0^{F2}\, e^{0.5(1-z-e^{-z})}}_{N_e^{F2}} + \underbrace{N_0^{P}\, e^{\left(\frac{-|h-h_{max}^{F2}|}{H^P}\right)}}_{N_e^{P}} \right) ds$$



with $z = \frac{h - h_{max}^{F2}}{H^{F2}}$

N_0^{F2} : F2 peak electron density

N_0^{P} : Plasmasphere basis density

h_{max}^{F2} : Peak height

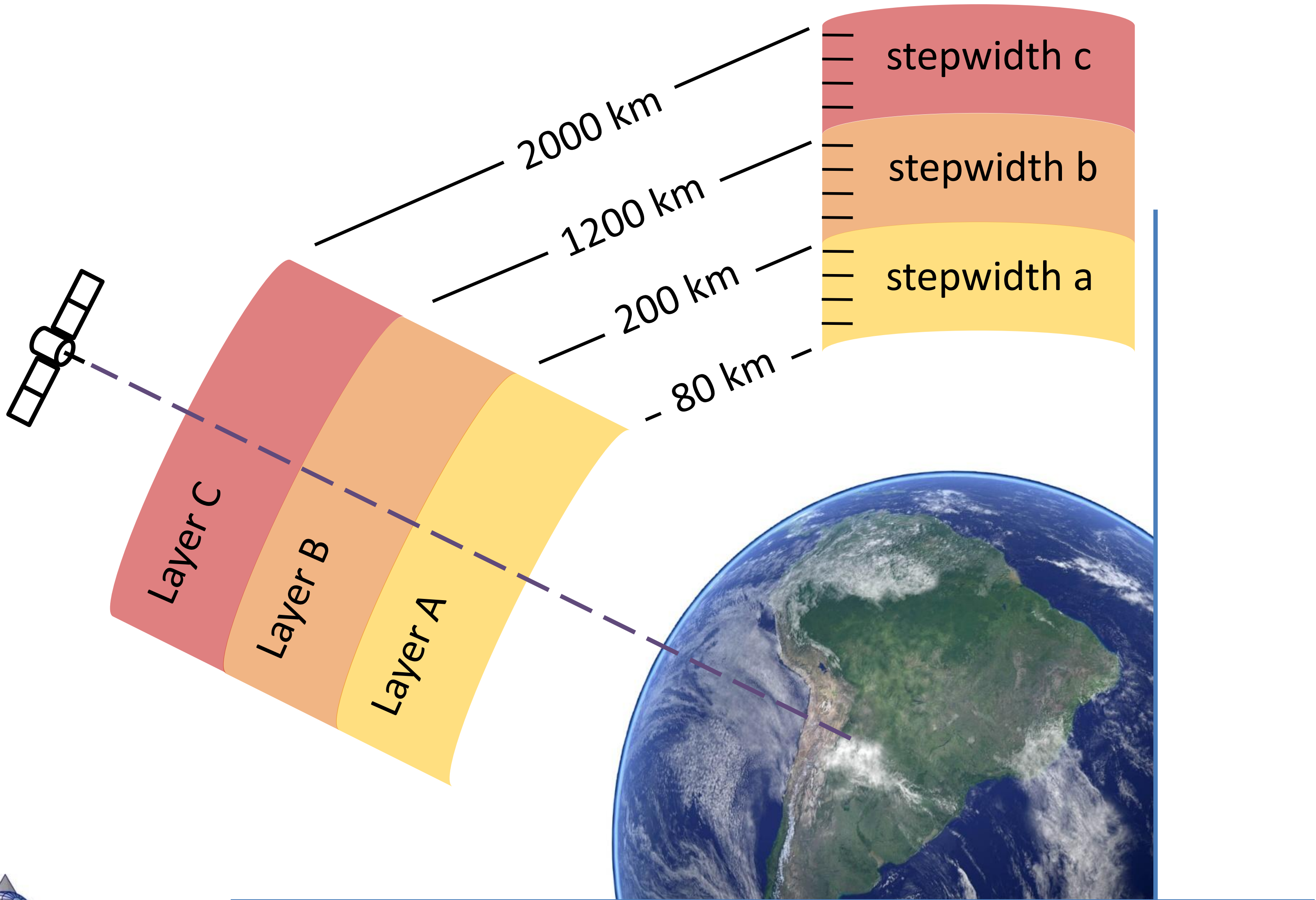
H^{F2} H^{P} : Scale height

h : Height

- The linearization of the derived Chapman/plasmasphere-function leads to the final observation equation for STEC which considers a background model and correction terms with respect to five unknown target parameters.

$$STEC(R,S,t)_{obs} = \underbrace{\boxed{\int_R^S \left(\overline{N_0^{F2}}\, e^{0.5\left(1-\bar{z}-e^{-\bar{z}}\right)} + \overline{N_0^{P}}\, e^{\left(\frac{-\left|h-\overline{h_{max}^{F2}}\right|}{\overline{H^{P}}}\right)} \right) ds}}_{\text{Background model}}$$

$$\underbrace{\boxed{+ \int_R^S \left(\Delta N_0^{F2} \frac{\partial N_e(h)}{\partial N_0^{F2}} + \Delta h_{max}^{F2} \frac{\partial N_e(h)}{\partial h_{max}^{F2}} + \Delta H^{F2} \frac{\partial N_e(h)}{\partial H^{F2}} + \Delta N_0^{P} \frac{\partial N_e(h)}{\partial N_0^{P}} + \Delta H^{P} \frac{\partial N_e(h)}{\partial H^{P}} \right) ds}}_{\text{Correction part}}$$


2000 km
1200 km
200 km
80 km
stepwidth c
stepwidth b
stepwidth a
Layer C
Layer B
Layer A

Pre-definitions

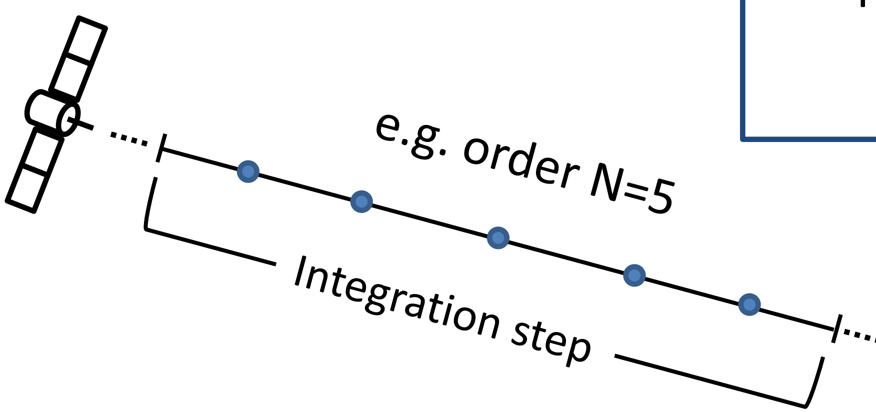
Layer boundaries

Integration stepsize

Certain number of integration steps due to all 3 layers

Quadrature order

Numerical Integration with N-point Gauss-Legendre Quadrature



$$I_f = \int_a^b f(x)dx \approx \sum_{i=1}^{N} f(x_i) w_i$$

- Objective: Approximation of an integral I_f as a sum of given sample values at nodal points x_i with positive-valued weights w_i

- Nodal points are derived from zeros $x_{1,\dots,}x_N$ of Legendre polynomials L_N and are required to calculate the corresponding weights

$$w_i = \int_{-1}^{1} \prod_{j=1, j\neq i}^{N} \frac{x - x_j}{x_i - x_j} dx > 0$$

- Gauss-Legendre quadrature formulas with N nodal points can integrate polynomials of degree $2N$ exactly

Hardware Information

- Ubuntu 11.04 Natty Narwhal
- 2048 MB Ram
- CPU: Intel Core P8400@ 2.26 GHz

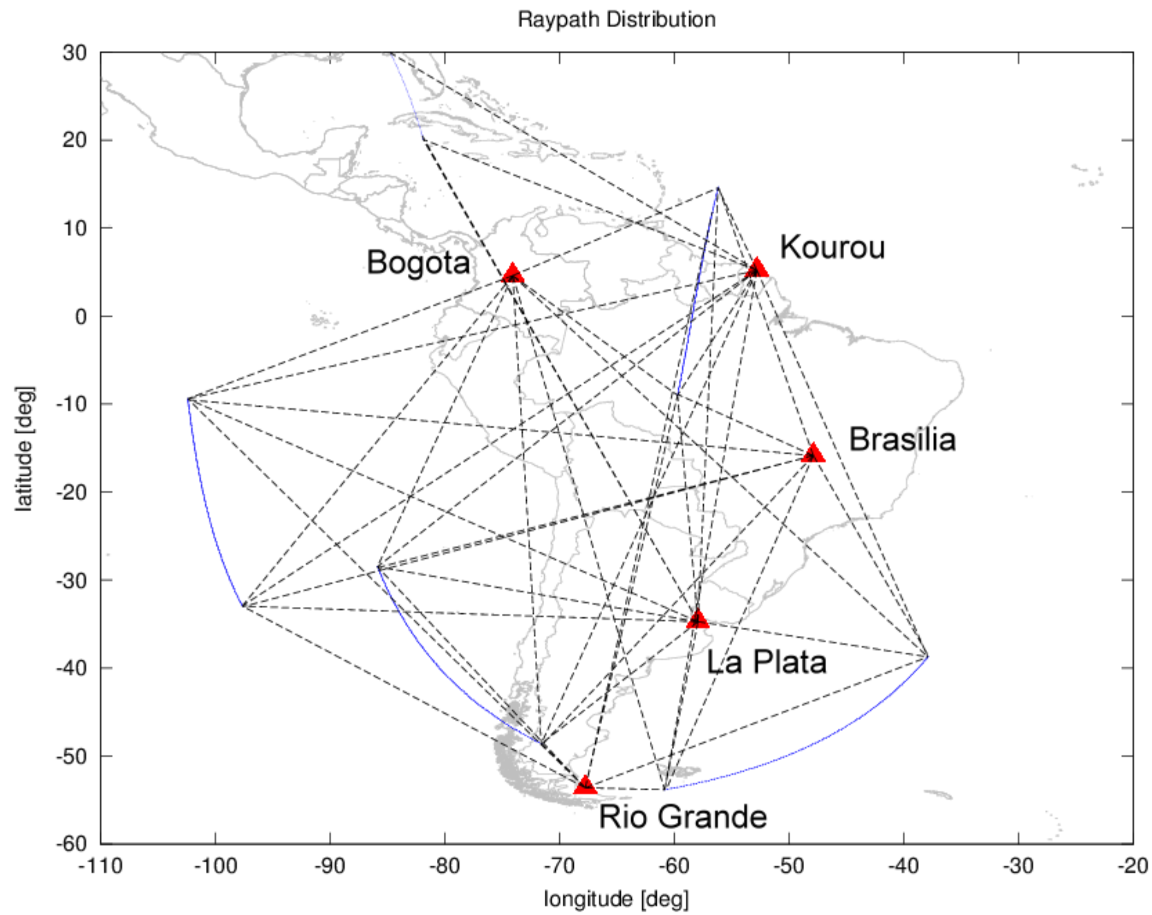
Software-Development

- Programming language: C++
- Math Libraries: Armadillo with LAPACK (open source), Standard Cmath

Reference solution

5 stations of „Sistema de Referencia Geocéntrico para Las Américas“ (SIRGAS)

- Integration stepwidth **1/1/1 km**
- Quadrature **order N=9** (9 weighted nodal points per integration step)
- Database: **06 – 07 UT**, 2002/07/01, 30s sampling rate, GPS only



Data processing

- Constant layer boundaries at 80/200/1200/2000 km height
- Pre-defined vertical Integration stepwidth – transformation to slant direction by

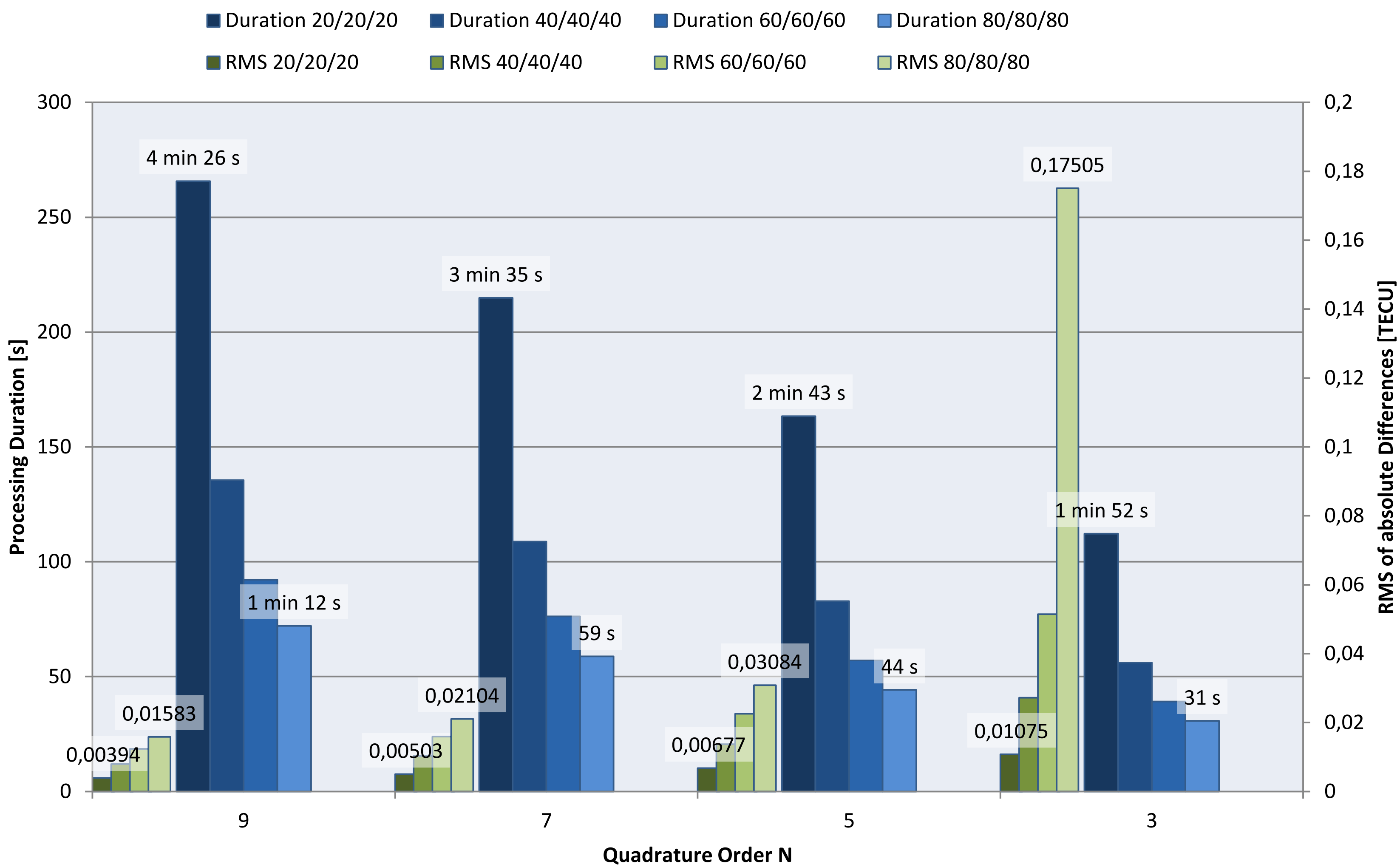
$$sSW \begin{cases} \frac{vSW}{\sin(el)} & : \quad 30° \leq el < 90° \\ \frac{vSW}{\sin(30)} & : \quad el < 30° \end{cases}$$

wherein

sSW : slant stepwidth
vSW : vertical stepwidth
el : elevation angle [deg]

- Varying quadrature order N (2,3,…,9)
- Focus on differences to calculated reference solution in comparison to the processing duration

RMS of reference data : 21.847 TECU



- An efficient processing strategy is required in order to deal with the complex formulas behind this ionospheric modeling approach

- … especially when different observation techniques such as GNSS, altimetry and LEO GPS will be considered in future… parallel computing!

- The Gauss-Legendre quadrature method has been succesfully applied

- Further tests with real observation data will show how the setting parameters (e.g. layer height, integration stepwidth and quadrature level) have to be chosen in order to yield a compromise between acceptable processing time and desired accuracy

- A 24 h GNSS data set (2002/07/01) w.r.t. to the 5 SIRGAS stations already covers 138,240 observations.
  Introducing vSW = 80/60/80 km and N = 5, the process duration yields 36 min.

Thank you for your attention

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