GPS time series analysis with Monte-Carlo singular spectrum analysis

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GIS

Outline . . .

- *Introduction of Singular Spectrum Analysis (SSA)*
- *How does Monte-Carlo SSA work?*
- *Summary*

Methodology

- Step 1 (Broomhead and King, 1986):
  - embed the time series $(x(t), 1 \leq t \leq N)$

$$D = \begin{pmatrix} x(1) & x(2) & x(3) & \cdot & \cdots & x(M) \\ x(2) & x(3) & x(4) & \cdot & \cdots & \cdot \\ x(3) & x(4) & x(5) & \cdot & \cdots & \cdot \\ \cdot & x(5) & x(6) & x(7) & \cdots & \cdot \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdot & \cdot & \cdot & \cdot & \cdot & x(N-1) \\ x(N-M+1) & \cdot & \cdot & \cdot & x(N-1) & x(N) \end{pmatrix}$$

  - compute the covariance matrix:

$$C_{BK} = \frac{1}{N-M+1} D^T D$$

Methodology Cont . . .

- Alternative way for Step 1 (Vautard and Ghil, 1989):
  - compute the autocovariance function of x_t

$$c_j = \frac{1}{N-j} \sum_{i=1}^{N-j} x_i x_{i+j}, \qquad 0 \leq j \leq M-1$$

  - form the Toeplitz matrix

$$C_{VG} = \begin{pmatrix} c(0) & c(1) & c(2) & \cdot & \cdots & c(M-1) \\ c(1) & c(0) & c(1) & \cdot & \cdots & \cdot \\ c(2) & c(1) & c(0) & \cdot & \cdots & \cdot \\ \cdot & c(2) & c(1) & c(0) & \cdots & \cdot \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdot & \cdot & \cdot & \cdot & \cdot & c(1) \\ c(M-1) & \cdot & \cdot & \cdot & c(1) & c(0) \end{pmatrix}$$

Methodology Cont . . .

- Step 2:
  - apply eigenvalue decomposition to C_{VG} and obtain EOFs

$$C_{VG} = E\Lambda E^T$$

  - compute principle components (PCs)

$$A = DE$$

Methodology Cont ...

- Step 3: group and reconstruct the signal (RCs)

$$x_i^k = \begin{cases} \frac{1}{i}\sum_{j=1}^{i} A_{i-j}^k E_j^k & 1 \leq i \leq M-1 \\ \frac{1}{M}\sum_{j=1}^{M} A_{i-j}^k E_j^k & M \leq i \leq N-M+1 \\ \frac{1}{N-i+1}\sum_{j=i-N+M}^{M} A_{i-j}^k E_j^k & N-M+2 \leq i \leq N \end{cases}$$

Chioce of lag-window size M

Some empirical suggestions:

- Vautard et al., (1992) suggested SSA succeeds in distinguishing of oscillations with periods between $M/5$ and M.
- Ghil et al., (2002) proposed one should balance two considerations: quantity of information extracted versus the degree of statistical confidence in that information.
- Rangelova et al., (2012) applied a three-year window (156 weeks) to retrieve long-term changes together with seasonal variations from weekly GRACE solutions.
- For weekly GPS time series, a two or three years window is proper enough for extracting annual and semi-annual signals. (Chen et al., Symposium, Shanghai, 2012)

Identify the signal

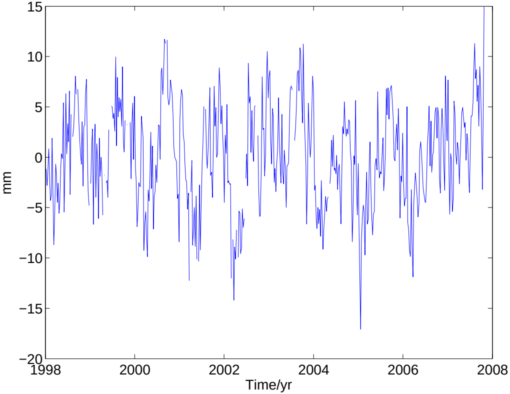
According to Plaut and Vautard (1994), harmonic oscillations can be identified in terms of:

- two consecutive eigenvalues are nearly equal
- the two corresponding time sequences described by EOFs are nearly periodic, with the same period and in quadrature
- the associated PCs are in quadrature

Real data analysis

Height direction of WIS1 station (window size $M = 157$)

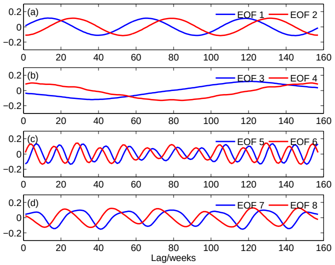
Original time series



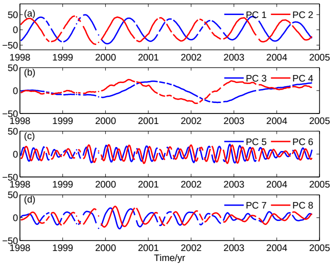
Real data analysis

Height direction of WIS1 station (window size $M = 157$)

EOFs



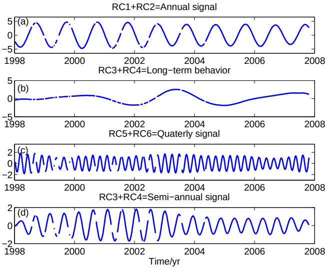
PCs



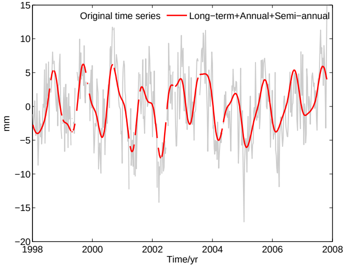
Real data analysis

Height direction of WIS1 station (window size $M = 157$)

RCs



Comparison



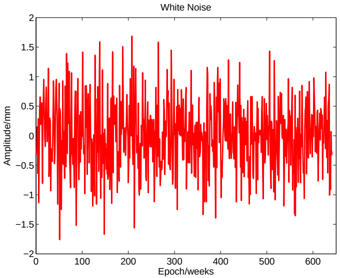
What's the problem with SSA when colored noise appears in the time series?

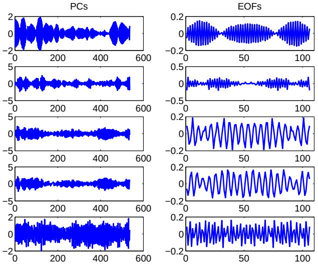
Problems with the interpretation of SSA

- Matrix-decomposition methods like SSA and EOF are based on the assumption that variance quantitatively reflects physical significance, which is simply not true for systems contaminated with coloured noise, nor for nonlinear systems in general. (Allen and Smith, 1996)
- If correlated noise is present in the data, artificial oscillations can be generated in the low frequency band of the eigen-spectrum and can mistakenly be taken as true signals. These oscillations can further enhance the spectral mixing of the trend and annual modes. (Rangelova et al., 2010, 2012)

Demonstration

Apply SSA to white noise

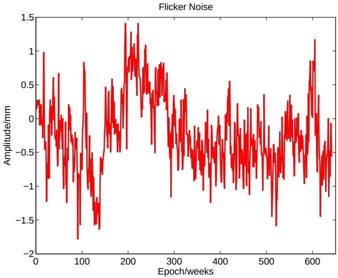


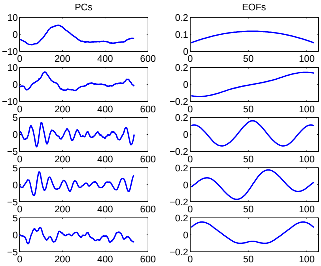


- Only high frequency oscillations are generated!

Demonstration

Apply SSA to flicker noise

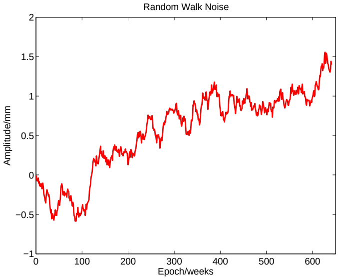


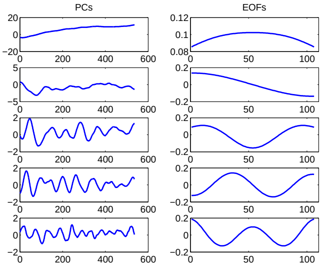


- Low frequency oscillations can be generated when SSA is applied to flicker noise!

Demonstration

Apply SSA to random walk noise





- Low frequency oscillations can be generated as well when SSA is applied to random walk noise!

How does Monte-Carlo test work?

Basic procedure

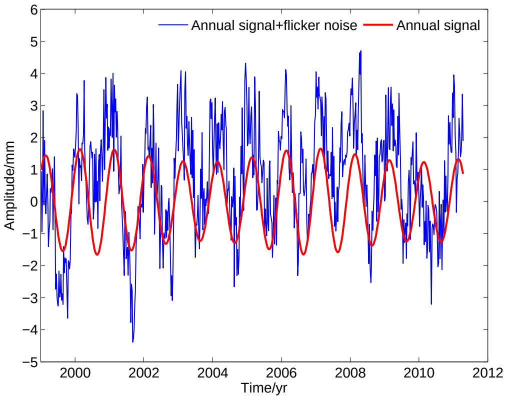
- Step 1: Apply SSA to the data and obtain EOFs (E_D) and Eigenvalues (λ_D)
- Step 2: Estimate the noise parameters and generate the surrogate realizations (Monte-Carlo simulation), calculate the covariance matrix of each surrogate realizations (C_R)
- Step 3: Project each surrogate realization onto the EOFs of the data and get the Eigenvalues (λ_R) of each surrogate

$$\lambda_R = E_D^T C_R E_D$$

- Step 4: Significance test with Eigenvalues λ_D and λ_R

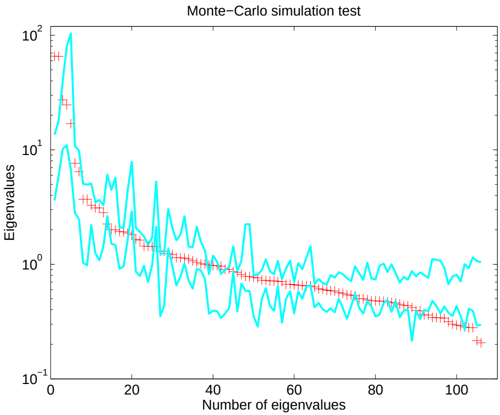
A simple simulation

Annual signal + Flicker noise (amplitude: 1mm)



A simple simulation

100 surrogate realizations, significance level p=0.95



Summary

- Singular spectrum analysis is useful for extracting seasonal signals from GPS time series.
- Artificial low frequency oscillations can be generated when time series is contaminated with colored noise.
- Based on simulation, MCSSA can be utilized for significance test against colored noise, like flicker noise in GPS time series

Thanks for your attention!