Hochgenaue regionale Geoidbestimmung durch die Kombination von terrestrischen und Satellitendaten

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2) Institute of Navigation

Graz University of Technology

Geodätische Woche 2014
Berlin, Deutschland
Introduction

- Current Austrian geoid initiative “Geoid for Austria - Regional gravity FIELD improved” (GARFIELD) - P25222-N29

- Combination of global gravity field models with terrestrial observations

- Discussed today:
  - *Effect of height differences between Digital Terrain Model (DTM) and gravity stations*
  - *Useful gravity observation groups for Variance Component Estimation*

*What is the magnitude of these effects for the geoid computation?*
Used Input Data Set (1)

- **Digital Terrain Model:** 176x196m
**Used Input Data Set (2)**

- **Measured gravity**: 71261 points
**Measured gravity:** 71261 points

- Digital Terrain Model: 176x196m

Differences max. ± 50m
Options for Investigation

• Where are the height differences coming from?
  - DTM heights or gravity station heights?

• How to correct?
  - 1\textsuperscript{st} Do nothing
  - 2\textsuperscript{nd} Fit DTM to station heights
  - 3\textsuperscript{rd} Fit station heights to DTM
Options for Investigation

- **Where are the height differences coming from?**
  - DTM heights or station heights?

- **How to correct?**
  - 1\textsuperscript{st} Do nothing
  - 2\textsuperscript{nd} Fit DTM to station heights
  - 3\textsuperscript{rd} Fit station heights to DTM

- **Changes in gravity reduction?**
- **Relative geoid changes?**
- **Absolute GPS/leveling validation?**

<table>
<thead>
<tr>
<th>Description input data set</th>
<th>Austrian gravity height quality [BEV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st} order leveling</td>
<td>&lt; 0.5 cm</td>
</tr>
<tr>
<td>2\textsuperscript{nd} order leveling</td>
<td>&lt; 2 cm</td>
</tr>
<tr>
<td>Triangulation</td>
<td>&lt; 20 cm</td>
</tr>
<tr>
<td>Elevation number</td>
<td>&lt; 200 cm</td>
</tr>
</tbody>
</table>
Gravity Station Height vs. DTM (1)

- Deviations between station height and DTM
  - Searching for nearest DTM point
Gravity Station Height vs. DTM (2)

- Deviations between station height and DTM
  - Searching for nearest DTM point
  - Compute difference between gravity station height and DTM
• Deviations between station height and DTM
  - Searching for nearest DTM point
  - Compute difference between gravity station height and DTM
  - Apply difference to DTM (only one single prism)
• **Remove-Compute-Restore** Technique

• Terrestrial input data
  - 71261 gravity measurements
  - 192 GPS/leveling observations (validation)

• Global gravity field model
  - GOCO03s [Mayer-Gürr T., et al. (2012)]

• Topographic reduction: **Prism formula**
  - DTM 176x196m
  - Standard crustal density of 2.670 kg/m³

• Computation: **Least squares approach**
  - Radial Basis Function parametrization
Remove – Let Heights Unchanged

<table>
<thead>
<tr>
<th>[mgal]</th>
<th>min</th>
<th>max</th>
<th>mean</th>
<th>rms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-46.64</td>
<td>39.98</td>
<td>-1.28</td>
<td>11.64</td>
</tr>
</tbody>
</table>

![Map of Austria with color gradient scale from -45 to 45 mgal]
Remove – Fit DTM to Station Heights

<table>
<thead>
<tr>
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<th>mean</th>
<th>rms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-47.17</td>
<td>38.85</td>
<td>-1.88</td>
<td>11.65</td>
</tr>
</tbody>
</table>
Remove – Changes in Reduction Step

<table>
<thead>
<tr>
<th>[mgal]</th>
<th>min</th>
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<th>mean</th>
<th>rms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-3.55</td>
<td>4.36</td>
<td>0.60</td>
<td>0.96</td>
</tr>
</tbody>
</table>

Differences caused by DTM changes
Restore – Relative Geoid Changes

<table>
<thead>
<tr>
<th>[cm]</th>
<th>min</th>
<th>max</th>
<th>mean</th>
<th>rms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-9.23</td>
<td>7.82</td>
<td>2.44</td>
<td>3.72</td>
</tr>
</tbody>
</table>

Geoid differences on 4x4 km grid
Restore – Absolute Validation (1)

<table>
<thead>
<tr>
<th>[cm]</th>
<th>min</th>
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<tbody>
<tr>
<td></td>
<td>-13.37</td>
<td>16.10</td>
<td>5.36</td>
</tr>
</tbody>
</table>

Fit DTM to station heights
Restore – Absolute Validation (2)

<table>
<thead>
<tr>
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<th>max</th>
<th>rms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-12.01</td>
<td>8.80</td>
<td>3.90</td>
</tr>
</tbody>
</table>

Unchanged DTM leads to a better result!
Unchanged DTM leads to a **better result**!

But a-posteriori $\sigma_{AUT}$ is **increasing** from 0.80 to 1.03 [mgal]
Building Observation Groups

• Situation
  - Only few absolute measurements
  - Huge amount of relative gravity
  - Data quality from neighbouring countries?

<table>
<thead>
<tr>
<th>Measurement system &amp; year</th>
<th>Austrian gravity quality [BEV]</th>
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<tbody>
<tr>
<td>Absolute gravity</td>
<td>&lt; 0.01 [mgal]</td>
</tr>
<tr>
<td>Relative gravity</td>
<td></td>
</tr>
<tr>
<td>LCR-D</td>
<td>&lt; 0.02 [mgal]</td>
</tr>
<tr>
<td>LCR-G</td>
<td>&lt; 0.02 [mgal]</td>
</tr>
<tr>
<td>LCR</td>
<td>&lt; 0.02 [mgal]</td>
</tr>
<tr>
<td>Scintrex CG3</td>
<td>&lt; 0.02 [mgal]</td>
</tr>
<tr>
<td>Worden 500</td>
<td>&lt; 0.10 [mgal]</td>
</tr>
<tr>
<td>Norgaard</td>
<td>&lt; 0.30 [mgal]</td>
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</tbody>
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FG-5
Scintrex
LaCoste & Romberg
• Situation

- Only few absolute measurements
- Huge amount of relative gravity
- Data quality from neighbouring countries?

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Building Observation Groups (1)

- Observation groups for Variance Component Estimation
  - 35595 LCR+Scintrex, 9339 Worden, 3816 Norgaard
  - 22770 from neighbouring countries
Restore – Relative Geoid Changes

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<tr>
<td></td>
<td>-2.45</td>
<td>0.94</td>
<td>-0.24</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Difference compared to “do nothing” solution!
Restore – Absolute Validation (3)

Improvements due to 4 observation groups!

<table>
<thead>
<tr>
<th>$VCE$ [mgal]</th>
<th>$\sigma_{LR}$</th>
<th>$\sigma_{Wor}$</th>
<th>$\sigma_{Nor}$</th>
<th>$\sigma_{N_{eigh}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.10</td>
<td>0.81</td>
<td>0.85</td>
<td>0.74</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<td></td>
<td>-12.53</td>
<td>8.27</td>
<td>3.70</td>
</tr>
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</table>
Building Observation Groups (2)

- Observation groups for Variance Component Estimation
  - 35595 LCR+Scintrex, 9339 Worden, 3816 Norgaard
  - 22770 from neighbouring countries – possible refinements?
Building Observation Groups (3)

- Observation groups for Variance Component Estimation
  - 35595 LCR+Scintrex, 9339 Worden, 3816 Norgaard
  - 22770 allocated for each neighbouring country
Minor improvements due to 10 observation groups!

VC\(E\) [m\(g\)al]

\(\sigma_{LCE} = 1.10\)
\(\sigma_{Wer} = 0.81\)
\(\sigma_{Nor} = 0.85\)
\(\sigma_{GER} = 1.06\)
\(\sigma_{SUI} = 1.12\)
\(\sigma_{ITA} = 1.04\)
\(\sigma_{SLO} = 0.98\)
\(\sigma_{HUN} = 0.18\)
\(\sigma_{SVK} = 0.55\)
\(\sigma_{CZE} = 1.87\)
Building Observation Groups (4)

- Observation groups for Variance Component Estimation
  - 35595 LCR+Scintrex, 9339 Worden, 3816 Norgaard – possible refinements?
  - 22770 allocated for each neighbouring country
Building Observation Groups (5)

- Observation groups for Variance Component Estimation
  - LCR+Scintrex, Worden, Norgaard and corresponding measurement epochs
  - 22770 allocated for each neighbouring country

17 observation groups
Institute of Theoretical Geodesy and Satellite Geodesy

Geodätische Woche 2014

Restore – Absolute Validation (5)

17 observation groups by introducing epochs! Best result!

V CE [mgal]

σ₁ = 0.86

σ₁₇ = 1.87

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<tr>
<td>-11.35</td>
<td>11.38</td>
<td>3.53</td>
<td></td>
</tr>
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Summary

• **Fit Digital Terrain Model to station heights**
  - Let heights to remain unchanged
  - Fit of DTM to station heights is not advisable

• **Individual weighting for Variance Component Estimation**
  - Solution is slightly improved due to different weighting schemes
  - 1 group Austria / 1 Neighbouring
  - 3 groups Austria / 1 Neighbouring
  - 3 groups Austria / 7 Neighbouring
  - 10 groups Austria / 7 Neighbouring

• **Absolute geoid validation**
  - Truth is not known - maybe still uncertainty in GPS/leveling observations?
  - The achieved results are close to the absolute error budget of < 3 cm
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Absolute Validation – Truth?

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<tbody>
<tr>
<td></td>
<td>-7.70</td>
<td>6.60</td>
<td>0.32</td>
<td>1.52</td>
</tr>
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Differences of 166 GPS/leveling observations period 2006-2010