Nutation coefficients and Earth rotation parameters determined from LLR data

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Introduction

Lunar Laser Ranging (LLR) has provided measurements of the Earth-Moon distance for more than 45 years. Over the years, the range precision has improved from the meter level of accuracy to the few millimetre level, under optimal conditions. The latter is possible, with the big telescope at Apache Point (Apache Point Observatory Lunar Laser-ranging Operation, APOLLO). Up to now LLR data are available from MLRS in Texas (USA), OCA in Grasse (France), APOLLO in New Mexiko (USA), Lure Observatory on MAUI/Hawaii (USA) and MLRO in Matera (Italy). The accuracy of the LLR model/analysis at our institute has been improved correspondingly to benefit from the high observational accuracy. With the updated analysis software package, several parameters of the Earth-Moon system, e.g. lunar orbit, coordinates of the observatories on Earth, the retroreflectors on the Moon and a multitude of relativistic parameters can be determined. Here, investigations of Earth orientation parameters are presented.

Nutation coefficients

Investigation of the effect of the transformation on the parameter determination

Characteristics of the fit

- Nutation coefficients A, B, A'', B'' in the equations

$$\Delta\psi_{MHB} = \sum_{i=1}^{n} (A_i + A'_i\, dt)\sin(\text{ARG}) + (A''_i + A'''_i\, dt)\cos(\text{ARG})$$

$$\Delta\epsilon_{MHB} = \sum_{i=1}^{n} (B_i + B'_i\, dt)\cos(\text{ARG}) + (B''_i + B'''_i\, dt)\sin(\text{ARG})$$

- estimated for four periods (18.6/9.3 years and 365.3/182.6 days)
- applying different modelling approaches for equinox based ITRS-GCRS transformation
- MHB2000 model of Mathews et al. [2002] for the nutation part
- precession modelled according to
  1. Fukushima [2003] and Williams [1994], denoted as FW
  2. Capitaine et al. [2003], denoted as P03

Comparison with MHB2000 model

- shows significant differences (see table 1) for both transformations
- especially for the long-periodic coefficients
- largest differences in longitude components A, A''

Differences depend on

- changing correlations, because of different implementation
- large correlation of 18.6 and 9.3 year periods
- unevenly distributed data (gaps in time series, Lunar orbit coverage, only few sites, weather, less accuracy in early years, ...)

Tab. 1: *Nutation coefficients for 18.6 and 9.3 year, 182.6 and 365.3 day periods. FW, P03 give the differences of our LLR fit to MHB2000 when using the respective implementation.*

Period		MHB2000 [mas]	FW [mas]	P03 [mas]
18.6 years	A	−17206.42	2.70 ± 0.20	5.21 ± 0.25
	B	9205.23	−0.48 ± 0.10	−1.32 ± 0.11
	A''	3.34	−4.62 ± 0.12	−3.46 ± 0.21
	B''	1.54	−2.29 ± 0.09	−2.19 ± 0.10
182.6 days	A	−1317.09	−2.38 ± 0.08	−1.69 ± 0.11
	B	573.03	0.25 ± 0.05	0.15 ± 0.05
	A''	−1.37	1.80 ± 0.07	1.85 ± 0.09
	B''	−0.46	0.23 ± 0.05	0.22 ± 0.05
9.3 years	A	207.46	0.45 ± 0.11	0.85 ± 0.18
	B	−89.75	−0.15 ± 0.07	−0.13 ± 0.08
	A''	−0.07	−1.50 ± 0.12	−0.97 ± 0.20
	B''	−0.03	−0.87 ± 0.08	−1.35 ± 0.09
365.3 days	A	147.59	−2.91 ± 0.10	**-0.51 ± 0.16**
	B	7.39	0.55 ± 0.06	**0.01 ± 0.07**
	A''	1.12	−2.30 ± 0.09	**-0.06 ± 0.11**
	B''	−0.19	−0.29 ± 0.05	**-0.02 ± 0.05**

Earth rotation parameters

Determination of pole coordinates x_p, y_p and Earth rotation phase ΔUT using data from APOLLO

Characteristics of the fit

- 5 normal points per night for time span 6/2006 – 9/2013 (182 nights) from APOLLO
- simultaneous determination of pole coordinates or Earth rotation phase and all coordinates of the observatories
- velocities of the observatories fixed to ITRF values
- a-priori EOP values from IERS C04 series, fixed for those nights that were not considered

Results for pole coordinates

- fitted values with their formal 1σ-errors given in figure 1
- accuracy 0.5 - 40 mas, less after 12/2010 due to reduced normal point accuracy
- correlation with coordinates of the observatories up to 20 - 40%, with each other 20 - 40%

Results for Earth rotation phase

- fitted values with their formal 1σ-errors given in figure 2
- accuracy 0.003 - 0.05 ms less after 12/2010 due to reduced normal point accuracy
- correlation with coordinates of the observatories up to 50%, with other ΔUT values 20 - 40%

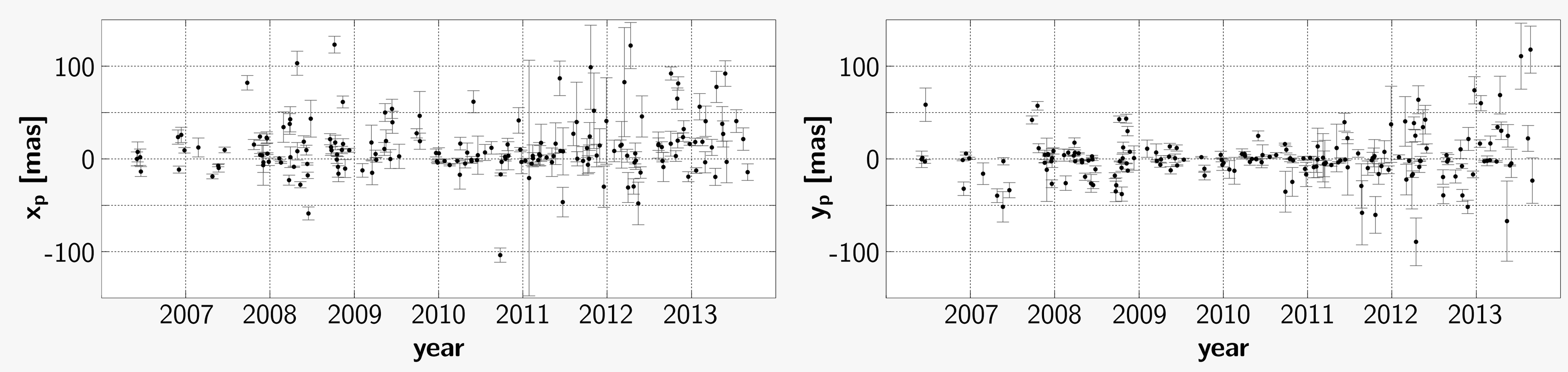

Fig. 1: *Pole coordinates fitted from normal points of APOLLO. Given are the determined values (dots) with the formal 1σ-errors.*

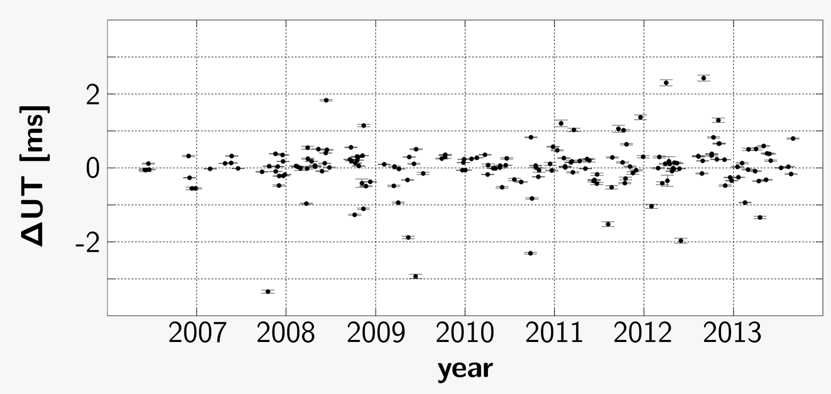

Fig. 2: *ΔUT fitted from normal points of APOLLO. Given are the determined values (dots) with the formal 1σ-errors.*

Conclusions

The determination of nutation coefficients depends on the modelled ITRS-GCRS transformation. Our investigations show that the realistic accuracy of nutation coefficients from LLR analysis is about 0.1 - 0.3 mas in obliquity and 0.2 - 0.5 mas in longitude. The LLR results for the nutation coefficients indicate a rotation of the analysis reference system and the GCRS. Future plan is a joined analysis of LLR and VLBI, to further improve the LLR analysis and to benefit from a combined solution for nutation.
Earth rotation parameters can be determined from LLR analysis, but the datum definition is difficult. The best LLR result is obtained for the Earth rotation phase ΔUT from APOLLO data, accuracy up to 0.003 - 0.05 ms. This result can be used for a combination of Earth rotation parameters from other geodetic techniques.

References and acknowledgement

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