On the capabilities of singular spectrum analysis for modeling geodetic time series

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1. Abstract

Separating different components, e.g. trend and seasonal signals, is of great importance in further interpretation of the geodetic time series. Following the successful application of SSA in modeling the time-variable GPS seasonal signals, this work aims to investigate the capabilities of applying SSA in other geodetic time series, e.g. lake level time series from satellite altimetry or basin scale water changes from GRACE.

2. Methodology

Singular Spectrum Analysis (SSA) is a non-parametric method that uses time domain data to extract information from short and noisy time series without prior knowledge of the dynamics affecting the time series. An important feature of SSA is that obtained trends are not necessary linear and extracted oscillations can be amplitude and phase modulated. In SSA, the main steps to derive the annual signals are as follows:

1. For a standard time series $x_t (t = 1, \cdots, N)$ and a maximum lag (or window size) M, a Toeplitz lagged correlation matrix C, i.e. its entries c_{ij}, depending only on lag $|i-j|$, is formed by

$$c_{ij} = \frac{1}{N - |i-j|} \sum_{t=1}^{N-|i-j|} x(t)x(t + |i-j|) . \qquad (1)$$

2. We apply eigenvalue decomposition to C and obtain eigenvalues λ_k and eigenvectors (also called EOFs) E_j^k of this matrix. E_j^k are sorted in descending order of λ_k, where indices j and k vary from 1 to M. The k^{th} principal component is

$$a_i^k = \sum_{j=1}^{M} x_{i+j} E_j^k , \qquad 0 \le i \le N - M . \qquad (2)$$

3. Each component of the original time series identified by SSA can be reconstructed, with the k^{th} reconstructed component (RC) series given by

$$x_i^k = \begin{cases} \frac{1}{i} \sum_{j=1}^{i} a_{i-j}^k E_j^k , & 1 \le i \le M-1 \\ \frac{1}{M} \sum_{j=1}^{M} a_{i-j}^k E_j^k , & M \le i \le N-M+1 \\ \frac{1}{N-i+1} \sum_{j=i-N+M}^{M} a_{i-j}^k E_j^k & N-M+2 \le i \le N . \end{cases} \qquad (3)$$

4. Grouping RCs with similar λ_k gives the desirable components, e.g. annual signals.

3. Aspects of applying SSA

Choice of lag-window size M

- The key parameter in SSA analysis;
- No universal rule for selection of the window size M;
- Basic rule: $M \le N/2$ due to the symmetry of the covariance matrix;
- General rule: to balance two considerations, i.e. quantity of information extracted versus the degree of statistical confidence in that information (Ghil et al., 2002);
- Several empirical rules and choices existing in the literature, e.g. two or three years window size for GPS time series (Chen et al.,, 2013).

Separability of signals

- The **w**-correlation between the RCs so as to define whether a weak separability or strong separability exists (Golyandina, 2013).

Unevenly sampled time series

- A modified SSA approach proposed by Schoellhamer (2001) for dealing with regularly sampled time series but with gaps;
- Resampling time series which are irregularly sampled, e.g. Tourian et al., (2015).

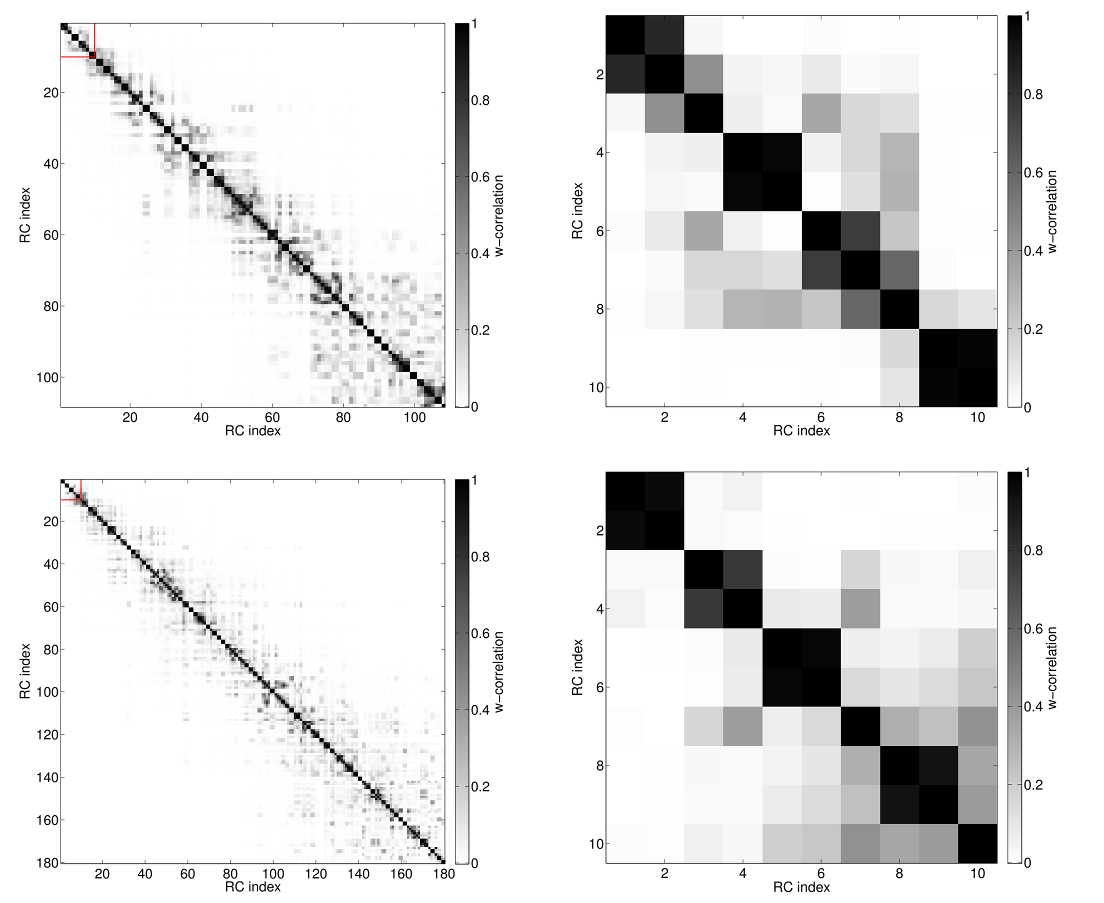

Figure 1: *w-correlation analysis of the water level time series using a 3-year window size ($M = 108$, top row), and a 5-year window size ($M = 180$, bottom row). The first ten RCs for each window size which are indicated by the red boxes are shown on the right column. Clearly, a 5-year window size provides better separability and it is preferable in practice.*

4. Application to lake level time series

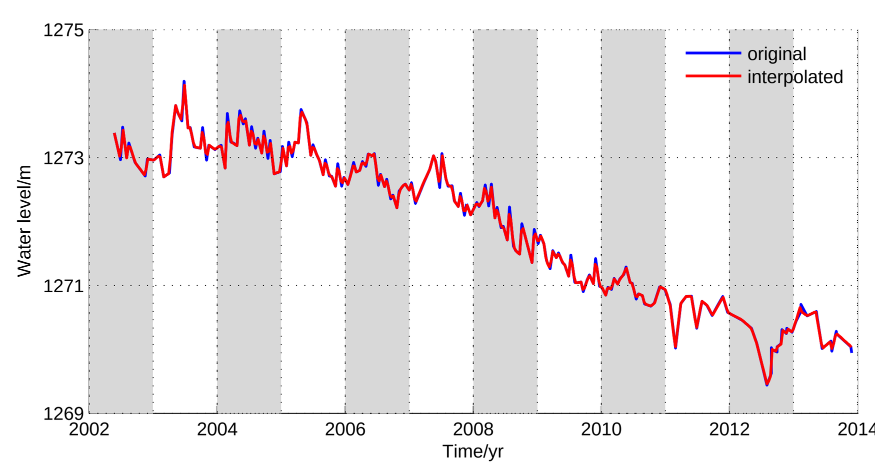

Figure 2: *The observed water level time series of Lake Urmia (Tourian et al.,, 2015) and the interpolated time series.*

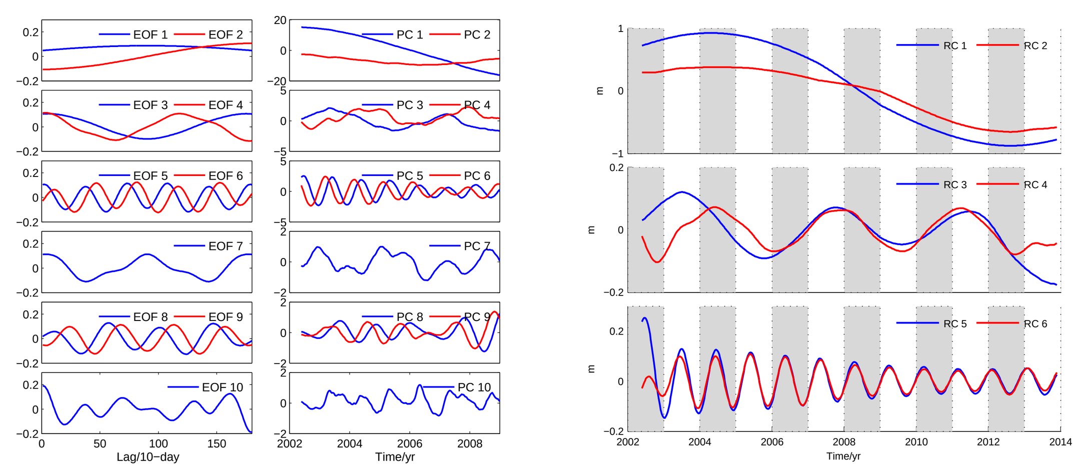

Figure 3: *Left: the first 10 EOFs and PCs are grouped in terms of the eigenspectrum; Right: the first six reconstructed components indicating non-linear trend, long-term seasonal signals and annual signals, respectively.*

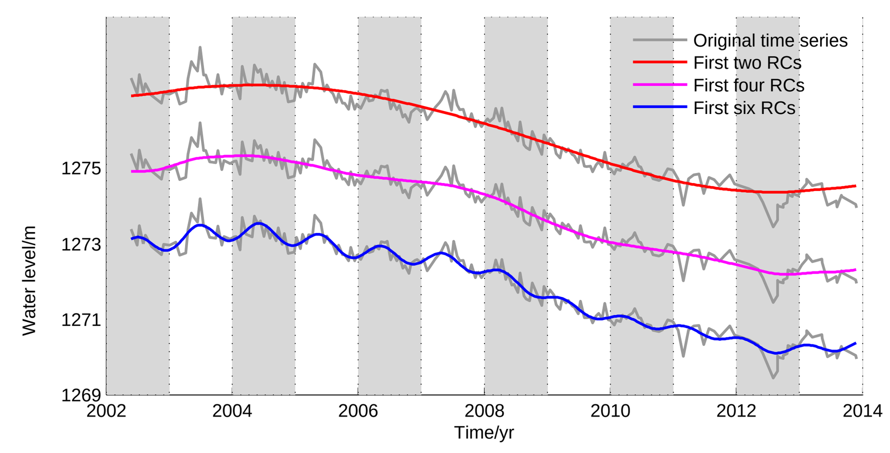

Figure 4: *Comparison of RCs with respect to the original time series.*

5. Application to basin averaged equivalent water height time series

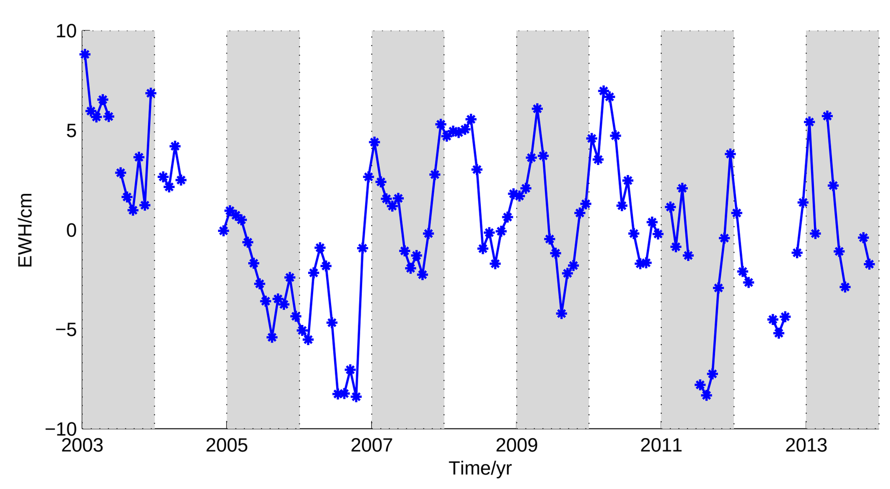

Figure 5: *The derived equivalent water height time series of the Congo Basin using GRACE products (GFZ RL05a).*

- Optimal window size $M = 60$ (5-year) determined via the **w**-correlation analysis;
- Schoellhamer (2001)'s approach was used to deal with gaps;

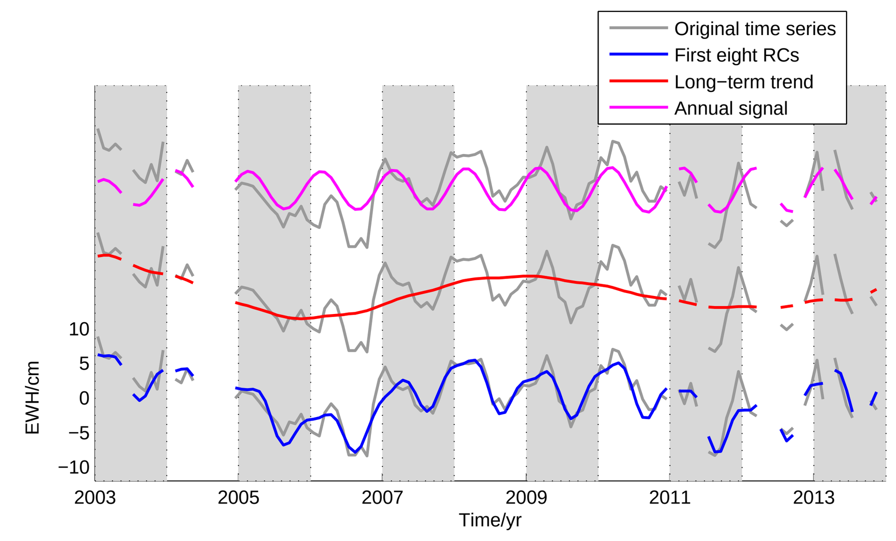

Figure 6: *Comparison of RCs with respect to the original time series in the Congo basin.*

6. Application to EOP time series

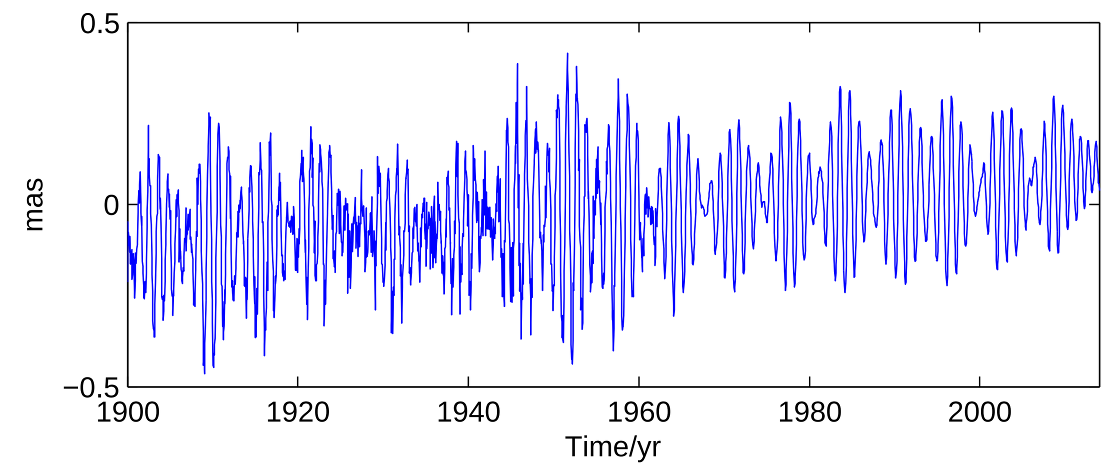

Figure 7: *Polar motion of x component (20 samples per year) from the EOP C01 solutons from 1900 up to 2014 (courtesy: IERS).*

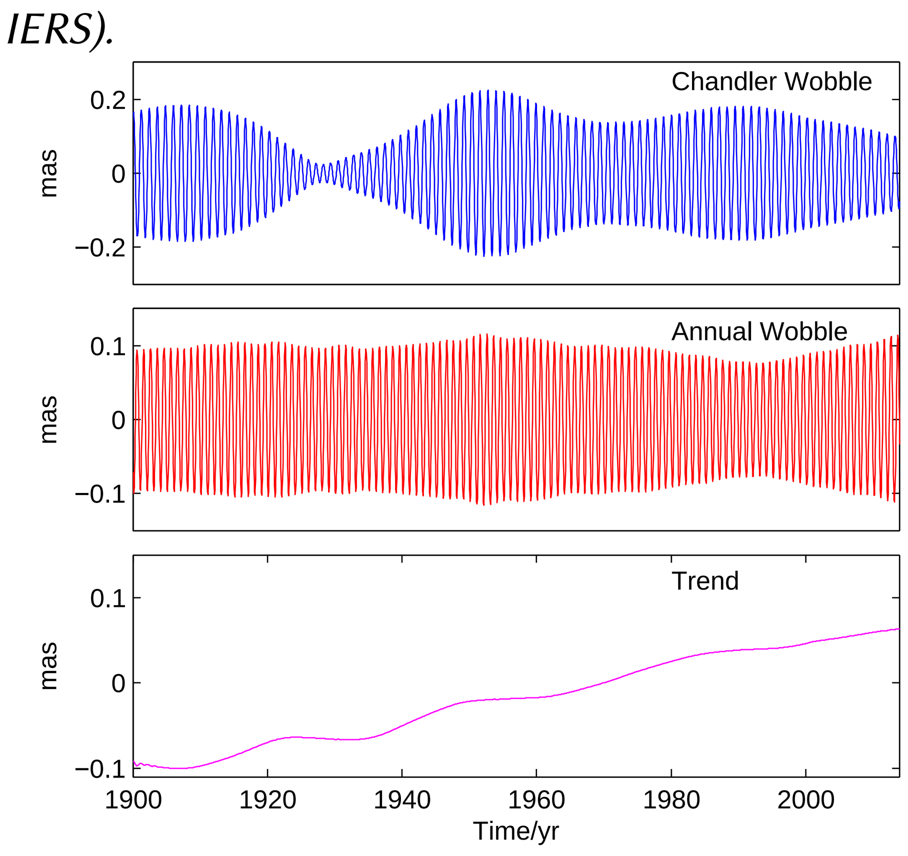

Figure 8: *Chandler wobble, annual wobble and non-linear trend are separated by SSA.*

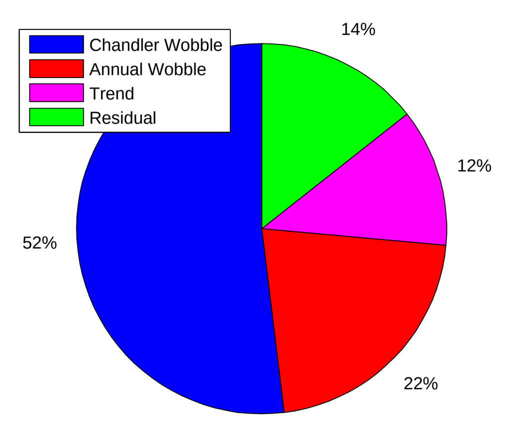

Figure 9: Contribution analysis.

- Optimal separability of Chandler wobble, annual wobble and non-linear trend at window size M=260 (13-year) determined by the **w**-correlation analysis as well as the Fourier frequency analysis.

7. Conclusions

- The **w**-correlation is helpful in determining the optimal window size M;
- SSA is a viable tool for analyzing and modeling the geodetic time series. It is not only useful for extracting seasonal signals but also non-linear long term trend behavior.

Acknowledgement

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